The Irish Journal of Education, 1989, *xxii*, 2, pp 51-64

MATHEMATICS ACHIEVEMENT IN IRISH PRIMARY SCHOOLS

Vincent Greaney
Educational Reserach Centre
St Patrick's College, Dublin

John Close
St Patrick's College Dublin

Studies of levels and correlates of mathematics achievement in Irish primary schools are reviewed In general students scored well in computation but poorly in geometry and problem solving In one of the surveys an international study of achievement in mathematics confined to content areas common to 12 participating countries or provinces Insh 13 year olds scored at about the mean Significant correlates of mathematics achievement include verbal ability achievement in Insh and English reading home background factors and gender

The realization during the 1960s in many western countries that school mathematics had failed to keep pace with developments in the field of mathematics and with the needs and demands of modern life led to pressure for curriculum reform As a result, substantial changes in content and method were proposed and adopted

In Ireland, many of the 'new maths' concepts were included in a curriculum for primary schools which was introduced in 1971 and reflected a child-centred approach to education (Ireland Department of Education, 1971) By comparison with its predecessor, the new curriculum placed greater emphasis on meeting children's interests and needs, concept development, practical work, and learning 'by doing' Among the aims of the curriculum were the stimulation of interest in mathematics, the development of understanding of mathematical structure and content, the development of proficiency in computation, and laying a foundation for further work in mathematics at post-primary level Emphasis was placed on the importance of guiding children to develop an understanding of concepts by manipulating materials and through exploration and discussion Computational practice was expected to follow, rather than precede, the formation of concepts The need for practical application and for integrating mathematics with other areas of the curriculum was also stressed

The mathematics section of the curriculum was broadened in lower junior and middle primary school classes to include measurement and geometry, areas that prior to this had been taught in senior grades only The scope of numbers

dealt with in lower grades was narrowed from 30 to 10 to allow for more careful development of concepts and principles In senior grades, factorization of algebraic expressions and congruence of triangle topics were eliminated Women were no longer excused from teaching algebra and geometry, as had been the case prior to 1971

Surveys of teachers (Conference of Convent Primary Schools, 1975, Irish National Teachers' Organisation, 1976) and of principals (Fontes & Kellaghan, 1977) have reported that the new curriculum was perceived to have resulted in an improvement in pupils' understanding of mathematical concepts and a deterioration in the standard of their memorization of number facts The findings of a survey of inspectors suggested that insufficient time was devoted to oral work and that too much emphasis was placed on the use of textbooks and workbooks (Ireland Department of Education, 1980)

In a study of parents in the mid-1970s, a sizeable minority expressed concern over their children's scholastic progress in arithmetic, a total of 29% regarded it only as 'fair' or 'poor' Comparable statistics for Irish reading and English reading were 33% and 15% respectively (Kellaghan, 1985)

ACHIEVEMENT IN MATHEMATICS

Three sources of information are available on achievement levels in mathematics since the introduction of the new curriculum These are a 1974 study of mathematics achievement carried out by the Educational Research Centre (Close, Kellaghan, Madaus, & Airasian, 1978, Kellaghan, Madaus, Airasian, & Fontes, 1976), the Department of Education's 1977, 1979, and 1984 national surveys (Ireland Department of Education, 1980, 1985), and the 1988 International Assessment of Educational Progress (IAEP) study (Lapointe, Mead, & Phillips, 1989) In some instances, it is difficult to compare survey results due to the use of different terminology and different groupings of objectives from survey to survey

The Educational Research Centre Survey

The Drumcondra Criterion-Referenced Mathematics Test (Educational Research Centre 1979) assessed performance on 55 objectives of the primary-school mathematics curriculum for fifth and sixth classes (Close et al , 1978) The objectives were inferred from the mathematics section of the Department of Education's *Teacher's handbook* (Ireland Department of Education, 1971) Two or three multiple-choice type questions were written for each objective Pupils who answered correctly a minimum of two out of two or two out of three

questions, depending on the objective involved, were deemed to have mastered that objective The test was administered to pupils in sixth class in primary school and first year in post-primary school (PP1) at the beginning and again towards the end of the school year Table 1 shows the mean percentage mastery levels by content areas On both occasions, at both levels, highest mastery was achieved on Operations with whole numbers (70% and 78% in sixth class, 70% and 75% in PP1, at the beginning and end of the school year respectively) and lowest on Arithmetic problems (24% and 36% in sixth class, 32% and 37% in PP1, at the beginning and end of the school year respectively) Separate analyses revealed that significant growth over the school year was registered on 53 objectives in sixth class and on 40 objectives in first year post-primary Mean

TABLE 1

MEAN PERCENTAGE MASTERY LEVELS BY CONTENT SECTION IN THE EDUCATIONAL RESEARCH CENTRE SURVEY

	Sixth Class		First Year Post Primary	
	Beginning of year	End of year	Beginning of year	End of year
Operations with whole numbers	69 75	78 00	69 75	75 00
Whole number concepts	53 70	64 40	58 20	65 60
Fraction concepts	52 20	64 80	56 50	62 20
Operations with fractions	46 60	59 10	47 90	52 70
Decimals and percentages	41 90	59 40	49 10	55 60
Algebra	40 30	52 70	53 80	60 30
Geometry	48 20	58 50	54 00	61 20
Charts and graphs	49 00	69 00	66 00	77 00
Arithmetic problems	23 80	36 00	32 00	36 00

Source Close et al 1978

growth in sixth class (11 8%) was approximately twice that in first year post-primary (6%) It is interesting to note that, for many objectives, percentage mastery levels at the beginning of post-primary schooling were lower than the corresponding levels at the end of sixth class in primary school The difference can be represented as an overall loss of 6% over the summer holidays

Objectives that proved particularly difficult in both years were ones dealing with multiplication and division of a fraction by a whole number, two-step algebraic equations, the distributive property, the least common multiple, the positioning of the decimal point, the conversion of percentages to fractions, and arithmetic word problems

Department of Education Surveys

The Department of Education's criterion-referenced mathematics tests were administered to nationally representative samples of second and fourth classes in 1977, and of sixth classes in 1979 and again in 1984 (Ireland Department of Education, 1980, 1985) A total of 26 objectives were tested in second class, 30 in fourth class, and 41 in sixth class Three questions were written for each objective A pupil who answered correctly at least two of the three questions was deemed to have mastered the objective

Results for second class show that most of the objectives tested were mastered by approximately 60% or more of the pupils Subtraction was the main area of difficulty with relatively low levels of mastery (less than 50%) In fourth class, many objectives, including ones in the areas of measurement, decimals, and the solution of word problems, appeared to be difficult for children (less than 50% mastery levels) Girls generally achieved higher levels of mastery than boys in both second and fourth classes

In sixth class, where pupils were tested in 1979 and 1984, most pupils mastered objectives related to performance on Whole Numbers For example, in 1984, 93% were able to subtract multi-digit numbers and 84% were able to multiply by two-digit numbers A lower level of mastery (61%) was recorded in long division It is interesting to note that between 1979 and 1984 the percentage of boys attaining mastery of long division dropped from 72% to 58%, the corresponding percentage for girls remained constant at 64% over the same period Percentage mastery levels were relatively high for most objectives involving Fractions and Decimals For example, in 1984, 78% of sixth class pupils could add or subtract two fractions while 76% could convert fractions to decimals and vice versa However, only about one in three pupils was able to position fractions on a number line Percentages of pupils mastering objectives relating to Geometry and Measurement were relatively low in both 1979 and

1984, most were in the 10% to 40% mastery range Boys outscored girls on most of these objectives

In general, there was little change in the level of mathematics performance in sixth class between 1979 and 1984, apart from the substantial improvement achieved on objectives relating to fraction concepts Performance on topics introduced during fifth and sixth class (factors and multiples, percentages, the circle, algebra, area, co-ordinates) was low relative to performance on objectives involving topics covered in earlier grades in the primary school The reports concluded with a recommendation for greater emphasis on concepts, the metric system, structured approaches to teaching problem-solving, and on regular revision and assessment

The 1988 IAEP Survey

The most recent evidence of pupils' mathematics achievement comes from an international study, the International Assessment of Educational Progress (Lapointe, et al, 1989) Testing was carried out in February, 1988, and was confined to 13-year-old students While approximately 90% of the Irish students were in post-primary school at the time of the survey, the data provide a useful indicator of primary-school achievement as almost all of the content should have been covered while the pupils were at primary school

The items for the test were selected from the larger pool of items used in the 1986 United States National Assessment of Educational Progress Through a consensus process involving participating countries, a pool of items considered appropriate was selected The process of item selection meant that certain aspects of the Irish primary-school mathematics curriculum were not represented in the test These included profit and loss, simple interest and VAT, co-ordinates, symmetry, and indices

Table 2 provides a summary of the performance of the Irish 13-year-olds on the six major content areas tested in the study Their overall performance on the 62-item mathematics test was significantly below the performance of students in Korea and in four Canadian provinces (Quebec-French, British Columbia, Quebec-English, New Brunswick-English), on a par with student performance in two further Canadian provinces (Ontario-English, New Brunswick-French), in Spain, and in the United Kingdom, and superior to student performance in one Canadian province (Ontario-French) and in the United States The study also revealed that, compared to students in the other samples, Irish students tended to spend a relatively large amount of time listening to the teacher or working alone in class and comparatively little time working in groups

Performance on items relating to Data Organization and Interpretation (48% mastery), Measurement (55%), and Geometry (56%) was low Somewhat better performance was recorded on Numbers and Operations (68%), Algebra, Relations, and Functions (72%), and Logic and Problem-Solving (72%) though it was still in the lower half of mean performance levels of the 12 participating countries and provinces

TABLE 2

PERFORMANCE OF IRISH 13 YEAR OLDS ON THE CONTENT AREAS TESTED IN THE IAEP SURVEY

Content Area	No of items	Ranking (among all 12 populations)	Average % correct	Range (among all 12 populations)
Numbers and Operations	24	9th	68	61 79
Algebra Relations and Functions	6	9th	69	60-80
Geometry	8	8th	56	47 72
Measurement	10	10th	55	44 71
Data Organisation and Interpretation	6	12th	48	48 75
Logic and Problem Solving	8	9th	72	63 78

Source Based on data in Lapointe et al 1989

It is not entirely appropriate to contrast levels of performance on similar items in the Department of Education and IAEP tests Apart from the fact that the tests were administered to different age groups, the scoring procedures were different In the case of the former, results were reported in the form of the percentages of pupils answering correctly at least two out of three items on a particular objective, whereas IAEP scores were based on the total number of items answered correctly

The 69 items in the IAEP test and the 123 items in the Department of Education test were classified by content and cognitive levels by two independent raters (see Tables 3 and 4) Identical classifications were recorded for 85% of the items, indicating a high measure of inter-rater reliability Following discussions between raters, agreement was reached on the classification of 95% of the items

The IAEP test devoted 50% of its items to whole numbers, fractions, and decimals, in contrast, 65% of items in the Department of Education test dealt with these topics (Table 3) A total of 20% of the IAEP items were devoted to measurement as opposed to 12½% of the Department of Education items

More importantly, in contrast with the Department of Education test, the IAEP test devoted three times the percentage of items to higher level skills (i e , routine and non-routine applications and problems) In addition, 30% of the IAEP items tested recall of facts and procedures while 40% of the items on the Department test did so (Table 4) An examination of the relative importance accorded to the different content areas in the most popular mathematics textbooks used in fifth and sixth classes in Irish primary schools led to the conclusion that the emphasis given to the various content areas in the textbooks was reflected more closely in the Department test than in the IAEP test

TABLE 3

APPROXIMATE PERCENTAGES OF ITEMS MEASURING CONTENT AREAS IN IAEP AND DEPARTMENT OF EDUCATION TESTS

Content Area	IAEP	Department of Education
Whole Numbers	30	25
Fractions and Decimals	20	40
Geometry	10	12 5
Measurement	20	12 5
Graphs and Statistics	10	5
Algebra and Relations	10	5

TABLE 4

APPROXIMATE PERCENTAGES OF ITEMS MEASURING DIFFERENT COGNITIVE LEVELS IN IAEP AND DEPARTMENT OF EDUCATION TESTS

Cognitive Level	IAEP	Department of Education
Recall of facts and procedures	30	40
Comprehension of concepts and principles	25	45
Routine applications and problems	30	15
Non routine applications and problems	15	0

Tables 3 and 4 above differ somewhat from the table reported in the IAEP Technical Report (Martin 1989) Numbers and Operations which was a single content area in the IAEP Table has been divided into two sections above (i) Whole Numbers and (ii) Fractions and Decimals The two cognitive process categories of (i) knowledge and (ii) skill included in the IAEP Table have been merged into one category Recall of facts and procedures

We wish to acknowledge the assistance of Catherine Mulryan St Patrick's College Dublin in the preparation of Tables 3 and 4

CORRELATES OF ACHIEVEMENT

Ability, English, Irish

Pupils who tend to perform well in mathematics also tend to do well on tests of English and Irish reading and verbal ability Data reported in the manuals for the Drumcondra Attainment Tests, Form A (Educational Research Centre, 1977, 1978a, 1978b) show that the correlations between achievement in mathematics and Irish reading range from 62 to 70, between achievement in mathematics and English reading from 67 to 71, and between achievement in mathematics and verbal ability from 65 to 74 (Table 5)

TABLE 5

CORRELATIONS BETWEEN MATHEMATICS AND ENGLISH READING IRISH READING AND VERBAL ABILITY IN THIRD TO SIXTH CLASSES

Mathematics	Verbal ability	English reading	Irish reading
Standard 3	65	67	62
Standard 4	67	71	69
Standard 5	75	71	67
Standard 6	74	69	70

Source Educational Research Centre 1977 1978a 1978b

The results of one small-scale study also found evidence of a strong relationship between personal attributes of pupils and their level of mathematics achievement (Greaney & Hegarty, 1982) The most important predictors of mathematics achievement, identified from a range of variables which included pupil sex, socio-economic status, and achievement press in the home, were verbal-reasoning ability (r = 72) and levels of achievement in English (r = 70) and Irish reading (r = 59) In this case, achievement press referred to parents' aspirations for the child and their interest in, knowledge of, and standards of reward for the child's educational achievement

Fontes (1977) examined four factors associated with problem-solving ability in mathematics pupils' verbal-reasoning ability, achievement in reading, knowledge of mathematical concepts, and knowledge of mathematical computation The study, which involved pupils in second, third, fourth, and fifth classes in primary schools, concluded that each of the four factors was significantly related to problem-solving at each class level Performance on the

problem-solving test, however, was more closely associated with performance on the two mathematical tests, concepts (r = 77), and computation (r = 71) than with verbal ability (r = 63) or reading achievement (r = 61)

Gender

Three studies have reported on gender differences in mathematics achievement In the first, first year post-primary boys performed significantly better than girls on most of the objectives in a criterion-referenced test based on the primary-school mathematics curriculum (Kellaghan et al, 1976) The results of the second study, which involved fifth-class pupils (N 138), showed that girls fared less well than boys on a test of the understanding of mathematical concepts (Greaney & Hegarty, 1982) The differences between boys and girls on the computation and problem-solving tests, however, were not significant Finally, in the large-scale IAEP international study, Irish 13-year-old boys and girls performed at about the same level of achievement on the overall test (Lapointe et al , 1989)

Home Background

To what extent does a child's home background have a bearing on mathematical performance? In a study of fifth-class pupils, socioeconomic background (measured on a 5-point scale based on father's occupation) correlated slightly but significantly (r = 18) with scores on a test of mathematical computation (Greaney & Hegarty, 1982) It was also found that performance was significantly related to the mother's level of education (r = 25) but not to the father's level Home background, when measured by level of emphasis on achievement (achievement press) in the home, correlated 34 with performance on the computation test. In an earlier study, with 8- and 9-year-olds in a disadvantaged area of Dublin, Kellaghan (1977) found significant correlations (ranging from 49 to 55) between six home-environment variables (achievement press, language model, academic guidance, family activeness, intellectuality, and work habits) and performance on a mechanical arithmetic test These results support the view that what parents *do* with their children in the home is more important than what they *are* (Bloom, 1976)

Homework

One would expect that appropriate homework assignments, that is, assignments directed towards material covered in class, would help to reinforce concepts and skills acquired in class In the IAEP study, students were asked to indicate if they did less than one hour, one to two hours, or three or more hours

of mathematics homework in a week (Lapointe et al , 1989) Irish 13-year-olds appear to devote less time to mathematics homework than students in any of the other eleven samples A total of 64% indicated that they spent less than one hour, 23% spent one to two hours, and 14% spent three or more hours at homework At the other extreme, the corresponding percentages for Korean students (the highest scoring sample) were 32% (less than one hour), 45% (one to two hours), and 23% (three or more hours)

One Irish study reported a correlation of - 17 between time devoted to homework and mathematics achievement (Greaney & Hegarty, 1982) A closer analysis of the data showed that those pupils who devoted an average amount of time (i e , 120-135 minutes) to homework over a four-day period tended to score highest Those who devoted most time to homework (210 minutes +) tended to score lowest while those who spent least time (less than 60 minutes) scored second lowest (of five categories) In this instance, the relatively low correlation between these two variables may be attributed to the fact that the 'weakest' pupils tended to devote most time to homework

Self-concept

A total of 49% of the Irish students who participated in the IAEP study considered themselves good at mathematics (Lapointe et al , 1989) The significance of this positive self-perception is not altogether obvious, however, since 68% of US students (who scored lowest among all participating populations) rated themselves good at mathematics while only 23% of Koreans (who scored highest) did so Thus the relationship between self-concept and mathematics achievement appears to be unclear in international comparative studies In a study of the self-concepts of sixth class pupils, Kellaghan and Fontes (1988) found that boys rated themselves more highly than did girls on their mathematical ability, as well as on their abilities in a range of other curricular areas

DISCUSSION

The surveys which we have reviewed suggest that Irish pupils perform well on computational skills but relatively poorly in geometry and problem-solving Studies highlight the strong relationship that exists between mathematics achievement on the one hand and pupil verbal ability and achievement in other curricular areas on the other A number of other significant (though weaker) predictors of mathematics achievement include home-background characteristics and gender

Performance on the mathematics test used in the international comparison of the mathematics achievements of 13-year-olds shows that Irish pupils score at about the mean for the 12 national and provincial participants In evaluating the significance of this finding it should be borne in mind that the IAEP test was limited to content areas shared by all 12 participants Some areas of the Irish senior primary-school curriculum were not tested while other areas which receive substantial emphasis were accorded little emphasis in the IAEP test This fact affects comparison of total scores across countries It is more interesting and useful to compare scores for specific areas of the curriculum

Analysis of performance in specific areas suggests that Irish pupils are faring relatively poorly in a number of important areas, notably Data Organization and Interpretation, Measurement, and Geometry and in tasks involving higher level mathematical understanding and comprehension In general, Irish pupils performed relatively poorly on questions that required abstraction or the presentation of mathematical tasks in non-routine formats A cursory examination of the most widely used textbook in sixth class in primary schools suggests that Irish pupils should have been exposed to many of the items on which they performed relatively poorly Given the emphasis placed on fractions and decimals in the Irish curriculum, it was particularly surprising to note that almost one in two were unable to write a fraction such as 7/100 as a decimal

In the Department of Education's national surveys, Irish pupils performed well on items such as 4 2/3 x 1 5/7 In the IAEP test, only one item was devoted to this area, whereas about 10% of the items on the Department's test were However, we might well ask how relevant these skills are today with the advent of the calculator Despite the fact that the calculator is very widely used in Irish society, it appears to have received very little attention in mathematics textbooks and has been ignored in formal national assessments of mathematics achievement Some teachers may be concerned that pupils' computational skills might be negatively affected by its introduction On this latter point, The Committee of Inquiry into the Teaching of Mathematics in Schools (1982) advised that 'from all the studies, the weight of evidence is strong that the use of calculators has not produced any adverse effect on basic computational ability' (p 110) Also, recent trends suggest that paper-and-pencil computations are becoming less important, while the estimation skills and understanding required to make proficient use of calculators (and computers) are becoming more important (National Council of Teachers of Mathematics, 1989)

The IAEP results suggest that too little attention may be devoted to mathematical tasks presented in a somewhat unfamiliar format and to questions involving abstraction or graphical representation While we have no direct

evidence on the level of emphasis placed on this aspect of mathematics in the classroom, the level of pupil performance on the IAEP and the content covered in national formal assessments of mathematics achievement suggest that there has been a tendency to stress the familiar or routine aspects of mathematics At this stage, a strong case can be made for greater classroom emphasis on open-ended, non-routine type problems, and on mathematical tasks which require pupils to represent situations numerically, graphically, verbally, or symbolically

Test results on the related areas of Charts and graphs and Data Organization and Interpretation highlight an interesting point Data from the national surveys indicate that sixth-class pupils achieved relatively satisfactory levels of performance in the area of Charts and graphs For example, of the seven major content areas listed by Close et al (1978), Charts and graphs recorded the second highest mastery level In the IAEP test, however, Irish pupils ranked last of twelve on the Data Organization and Interpretation items We feel that the difference in ranking reflects characteristics of the items used to measure pupils' knowledge of pictorial representation of data in the two studies The items used in the IAEP test measured a higher level of understanding of pictorial representation than did those used in the Irish tests

The present review demonstrates the value of national and international surveys in providing objective information about curriculum and instruction issues in primary-school mathematics Such surveys can provide useful information about strengths and weaknesses in students' achievements (and by implication in the curriculum) over time and compared to the achievements of students in other countries One weakness in the studies is that they do not include the assessment of practical skills Thus, we do not have any evidence about Irish primary-school pupils' level of mastery in practical mathematics, in tasks such as measuring distance weight, volume, data recording, estimation, and construction of graphical representation of data The emphasis given to practical work in the official curriculum is not likely to be reflected in practice as long as assessments do not take it into account

REFERENCES

Bloom, B S (1976) *Human characteristics and school learning* New York McGraw-Hill

Close, J S , Kellaghan, T , Madaus, G F , & Airasian, P W (1978) Growth in mathematical attainments of pupils *Irish Journal of Education*, *12*, 3-21

Committee of Inquiry into the Teaching of Mathematics in Schools (1982) *Mathematics counts* (Report prepared under the Chairmanship of Dr W H Cockcroft) London Her Majesty's Stationery Office

Conference of Convent Primary Schools in Ireland (1975) *Evaluation of the new curriculum in primary schools* Dublin Author

Educational Research Centre (1977) *Drumcondra Attainment Tests, Level III Form A* Dublin Author

Educational Research Centre (1978a) *Drumcondra Attainment Tests, Level I, Form A Mathematics and English Administration and Technical Manual* Dublin Author

Educational Research Centre (1978b) *Drumcondra Attainment Tests, Level II Form A Mathematics, Irish and English Administration and Technical Manual* Dublin Author

Educational Research Centre (1979) *Drumcondra Criterion-Referenced Mathematics Test, Level 6* Dublin Author

Fontes, P J (1977) Reading ability and mathematics In V Greaney (Ed), *Studies in reading* Dublin Educational Company

Fontes, P J , & Kellaghan, T (1977) *The new primary school curriculum Its implementation and effects* Dublin Educational Research Centre

Greaney, V , & Hegarty, M (1982) Factors related to mathematics achievement Paper presented to the Annual Conference of the Irish Mathematics Teachers' Association, Dublin

Ireland Department of Education (1971) *Primary school curriculum Teacher s handbook, Part I* Dublin Author

Ireland Department of Education (1980) *Tuairisc ar theagasc na matamaitice sna bunscoileanna (An dara heagran)* Dublin Author

Ireland Department of Education (1985) *Tuairisc ar an mhatamaitic i Rang VI* Dublin Author

Irish National Teachers' Organisation (1976) *Primary school curriculum Curriculum questionnaire analysis* Dublin Author

Kellaghan, T (1977) Relationships between home environment and scholastic behavior in a disadvantaged population *Journal of Educational Psychology, 69*, 754-780

Kellaghan, T (1985) The achievements of primary education in Ireland In Council of Europe (Ed) *Research into primary education* Lisse Swets & Zeitlinger

Kellaghan, T , & Fontes, P J (1988) Gender differences in the scholastic self-concepts of Irish pupils *Irish Journal of Education, 22*, 42-52

Kellaghan, T , Madaus, G F , Airasian, P W , & Fontes, P J (1976) The mathematical attainments of post-primary school entrants *Irish Journal of Education, 10*, 3-17

Lapointe, A E , Mead, N A , & Phillips, G W (1989) *A world of differences An international assessment of mathematics and science* Princeton, NJ Educational Testing Service

Martin, M O (1989) *IAEP study Technical report* Dublin Educational Research Centre

National Council of Teachers of Mathematics (1989) *Curriculum and evaluation standards for school mathematics* Reston, VA Author