The Irish Journal of Education, 1978, *xii*, 1, pp. 3-21.

GROWTH IN MATHEMATICAL ATTAINMENTS OF PUPILS*

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The performance of a sample of 1,167 sixth class and 640 first year post-primary pupils was assessed at the beginning and end of the school year using a test based on 55 objectives of the mathematics curriculum for fifth and sixth classes in primary school. Significant growth in the percentages of pupils mastering the objectives was achieved on 53 objectives during sixth class. During the first year of post-primary schooling, significant growth was achieved on 40 objectives.

Little information on the mathematical attainments of representative samples of Irish school students is available. While in countries such as the United States, a common source of information on scholastic achicvement is the performance of students on standardized norm-referenced tests (14), such tests have only recently been developed in Ireland, and as yet no information is available on trends over time in the performance of pupils on the tests. In recent years in the United States, information on attainment has been provided by the National Assessment of Educational Progress surveys (NAEP) (15), which describe the mathematical attainments of 9, 13, and 17 year-olds and adults, not in terms of norm-referenced scores, but in terms of six levels of mathematical skills. These data are gathered periodically to provide census-type information on the attainments of students over a period of time.

Criterion-referenced tests seem especially appropriate in examining levels of mastery of curriculum objectives (1, 5, 16). In a recent study of the mathematical attainments of Irish post-primary entrants (10), such a test was used. The test was based on 55 objectives of the mathematics curriculum for fifth and sixth classes in primary school and was administered to a sample of pupils in their first term in post-primary school. The percentage

* This study was supported by funds from the Carnegie Corporation of New York.
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of pupils mastering the objectives ranged from a low of 16 to a high of 92, pupil success was highest on objectives relating to operations with whole numbers and interpretation of charts and graphs and lowest on objectives relating to arithmetical problem solving The findings suggest that many children enter post primary school without having mastered the objectives of the primary school mathematics curriculum

The present study, by examining performance in both the final year of primary school and the first year of post primary school, broadens and extends the information base provided by the earlier investigation Here we may note that the syllabus covered during the first year in post-primary school (9) overlaps to a considerable extent with the content of the curriculum for the final years of the primary school (8) The data we report for the post primary sample's first testing are the same as those reported in the earlier study (10)* However, we go beyond the data contained in that study by examining changes in attainment over the course of the first year in post-primary school We also examine changes in the attainments of pupils in the final year of primary school Thus, for 55 objectives of the mathematics programme for fifth and sixth classes in the primary school, an attempt was made to obtain measures of the growth of a sample of pupils in their final year in primary school and of a sample in their first year in post primary school

MEHTOD

Sample

In 1974, teachers in a nationally representative sample of 30 primary schools agreed to administer the Drumcondra Criterion Referenced Mathematics Test (DCRMT) to all pupils in sixth class in the autumn of 1974 and again about six months later in the summer of 1975 Teachers in eight secondary and three vocational schools also administered the test to all first year pupils in the autumn and summer of the school year Information on the numbers of schools and pupils taking part in the investigation is presented in Table 1 As can be seen from the table, two primary, one secondary and one vocational school selected for the original sample did not participate, further, in participating schools, results from only one testing are available for some pupils This was due to a variety of

* There are some differences between the figures reported in the earlier study (9) and those reported for the first testing of post-primary pupils in the present study The reason for these differences is that data in the present study are based on the performance of pupils who took the test on two occasions

factors: absenteeism during one of the testing sessions, incomplete or incorrectly filled answer sheets, or change of school. Test scores for both autumn and summer were obtained for 1,167 sixth class pupils and 640 first year post-primary pupils. Analyses are based on the data obtained for these pupils.

TABLE 1

NUMBERS OF PARTICIPATING SCHOOLS AND PUPILS

		No. of schools selected	No. of schools returning tests	No. of pupils taking test in autumn and/or summer	No. of pupils taking test in both autumn and summer
6th Class Primary		32	30	1,390	1,167
1st Year	Sec.	9	8	785	486
Post-Primary	Voc.	4	3	195	156

Instrument

The Drumcondra Criterion Referenced Mathematics Test is based on the primary school mathematics curriculum for fifth and sixth classes in Irish primary schools (8). To develop the instrument, a list of curriculum objectives was derived from content analysis of the curriculum. The objectives were then reviewed and modified by a panel of mathematics teachers. The final set of 55 objectives was judged by the panel to represent a reasonably comprehensive list of the behaviours which would be demonstrated by pupils who had mastered the content of the curriculum for fifth and sixth standards. For all but one objective, two or three multiple choice items were written (for one objective, eight items were included). The test contains a total of 155 items. The main categories of objectives covered by the test are: (A) operations with whole numbers; (B) whole number structure; (C) fractional number structure; (D) operations with fractions; (E) decimals and percentages; (F) algebra; (G) geometry; (H) charts and graphs; (J) arithmetic problems. Thirty-five objectives involve comprehension, fourteen computation and six problem solving.

The tests were administered by the pupils' own teacher and returned to the Educational Research Centre for scoring. A number of models were reviewed for the a priori or posterior estimation of mastery performance levels (e.g., 3, 6, 13) but none appeared to be particularly appropriate for item sets made up of two or three items It was finally decided that a pupil would be regarded as having 'mastered' an objective if he or she answered correctly two items in the case of objectives for which either two or three items were included and six items in the case of the objective for which eight items were included This definition of mastery is *not* quite in keeping with a view of mastery as an all-or none description of learning (12) which would require 100% performance as an indication that mastery of the objectives had been attained However, taking into account expected errors of measurement, our procedure seems reasonable

RESULTS

Results are presented in two ways for sixth class and first year post primary pupils separately Firstly, for each of the 55 objectives, Table 2 provides information on the percentage of pupils who move from non-mastery to mastery (*a*), the percentage who move from mastery to non-mastery (*b*), the percentage who exhibit non-mastery on both occasions (*c*) and the percentage who exhibit mastery on both occasions (*d*) By summing *b* and *d*, one obtains the total percentage of pupils who achieved mastery at the beginning of the year Summing *a* and *d* gives the total percentage who achieved mastery at the end of the year Subtracting the former $(b + d)$ from the latter $(a + d)$ gives $a - b$, which is the net percentage growth associated with each objective during the school year

To obtain an estimate of the significance of differences between the number of pupils exhibiting mastery on each objective on the two occasions on which they took the test, the standard error of the difference between non independent proportions was calculated (11, pp 52ff) Net change in mastery for each objective was then divided by the standard error using the following formula

$$z = \frac{a - b}{\sqrt{\frac{a + b}{N}}}$$

where *a* is the proportion of pupils who went from non-mastery to mastery, *b* is the proportion who went from mastery to non-mastery and N is the total number of pupils A probability level exceeding 01 was set as one which would indicate statistical significance

Secondly, mean percentages achieving mastery for each of nine sections or groups of objectives were calculated and are presented in Table 3.

Sixth class

Table 2 contains the results for the beginning and end-of-year testings of sixth standard pupils. Across the 55 objectives, the average percentage of pupils who could be classified as masters entering sixth class was 47.04. (This figure is the total of the means for columns *b* and *d*.) At the end of the year, the average percentage of pupils classified as masters over the 55 objectives was 58.78. (This figure is the total of the means for columns *a* and *c*.) Net growth over the year $(a - b)$ was 11.8% over the 55 objectives with a range from 2 to 27 percent.

Across the 55 objectives, the average percentage of pupils exhibiting mastery on both occasions was 37.2. This does not mean that this percentage of pupils had mastered all objectives both times. What it means is that this percentage, on average across objectives, showed mastery at the beginning of the year and maintained it at the end of the year. Similarly across the 55 objectives, we can see that 31.4% of pupils failed to exhibit mastery on both occasions.

On every single objective, there was movement both from mastery to non-mastery and from non-mastery to mastery. For the former, the mean percentage movement across the 55 objectives was 9.9%, with a range from 5 to 16 percent. For the latter, the mean percentage movement was 21.6%, with a range from 7 to 28 percent.

A level of growth which was statistically significant was achieved for all objectives, with the exception of two – simple whole number operations with zero (B6) and problems involving value added tax (J3). Mean percentage growth was 11.8 with a range from 2 to 27.

To relate our data to the principal mathematical content areas of the curriculum, the mean percentage mastery per objective for each of nine major sections of the curriculum was calculated for beginning and end-of-year performances (Table 3). It is clear that, at both times of year, most students did not master objectives relating to arithmetic problem solving (J), while about half failed to master objectives relating to decimals and percentages (E) and algebra (F). Pupils performed best on objectives in the area of whole number operations (A), for which average percentage mastery per objective was 70% at the beginning of the year and 78% at the end.

TABLE 2

PERCENTAGES RELATING TO GROWTH IN THE MATHEMATICAL ATTAINMENTS OF SIXTH CLASS AND FIRST YEAR POST–PRIMARY PUPILS ON 55 OBJECTIVES

OBJECTIVES	CHANGE IN MASTERY PERCENTAGES							
	Non-mastery to mastery (*a*)		Mastery to non-mastery (*b*)		Non-mastery on both occassions (*c*)		Mastery on both occasions (*d*)	
	6th	1st Yr	6th	1st Yr	6th	1st Yr	6th	1st Yr
A Operations with Whole Numbers								
The student can								
1 Add a column of numbers containing not more than five digits	17	15	11	12	5	5	67	69†
2 Subtract two numbers containing not more than five digits	16	18	10	10	7	11	67	62
3 Multiply two numbers containing not more than five digits	16	18	12	15	12	13	60	54†
4 Divide numbers containing not more than three digits	25	19	9	13	23	23	44	45
B Whole Number Structure								
The student can								
1 Position a number on the number line	24	18	12	9	22	15	42	58
2 Complete simple number sequences	11	8	7	6	4	5	78	81†
3 Identify prime numbers and composite numbers	27	20	10	14	26	33	36	33
4 Identify the commutative property	28	22	11	10	29	27	32	41
5 Identify the distributive property	19	23	7	8	69	56	5	13

For each objective at each grade a measure of net percentage growth may be obtained by subtracting the figure in the Mastery to non-mastery' (*b*) column from the figure in the Non-mastery to mastery (*a*) column

TABLE 2(Contd.)

OBJECTIVES	CHANGE IN MASTERY PERCENTAGES							
	Non-mastery to mastery (*a*)		Mastery to non-mastery (*b*)		Non-mastery on both occasions (*c*)		Mastery on both occasions (*d*)	
	6th	1st Yr	6th	1st Yr	6th	1st Yr	6th	1st Yr
6. Perform simple whole number operations involving zero.	7	5	5	5	3	3	85†	88†
7. Factor two and three digit numbers.	19	13	8	10	16	13	57	64†
8. Identify common factors between two numbers.	24	17	13	9	21	26	43	48
9. Identify the highest common factor between two numbers.	22	18	14	13	25	26	39	44
10. Identify the least common multiple of two numbers.	25	24	11	11	43	47	21	18
C. Fractional Number Structure								
The student can:								
1. State a ratio as a fraction.	25	17	11	14	16	19	48	50†
2. State a fraction in a number of equivalent forms.	27	21	14	12	31	35	28	32
3. Reduce a fraction to its simplest terms.	23	13	7	8	20	23	50	55
4. Complete a ratio statement.	21	18	10	10	23	22	46	50
5. Sequence fractions.	20	19	11	13	48	43	21	26
6. Convert an improper fraction to a mixed number and vice versa.	19	11	7	8	14	19	61	61†

TABLE 2(Contd)

OBJECTIVES	CHANGE IN MASTERY PERCENTAGES							
	Non-mastery to mastery (*a*)		Mastery to non mastery (*b*)		Non mastery on both occasions (*c*)		Mastery on both occasions (*d*)	
	6th	1st Yr	6th	1st Yr	6th	1st Yr	6th	1st Yr
D Operations With Fractions								
The student can								
1 Add and subtract two fractions having the same denominator	13	11	7	11	6	8	74	70†
2 Add and subtract two fractions having different denominators	21	14	8	12	23	32	48	42†
3 Multiply two fractions having the same denominator	25	20	15	13	34	41	26	25
4 Multiply two fractions having different denominators	22	17	12	13	14	20	53	50†
5 Divide two fractions having the same denominator	26	16	9	14	20	30	45	40†
6 Divide two fractions having different denominators	26	16	8	9	34	41	32	33
7 Add or subtract three fractions having different denominators	21	13	8	10	31	35	41	41†
8 Subtract a fraction from a whole number and vice versa	22	17	9	11	37	34	32	39
9 Multiply a fraction by a whole number	21	15	6	7	56	59	17	20
10 Divide a fraction by a whole number and vice versa	18	15	7	9	65	66	10	10

TABLE 2(Contd.)

OBJECTIVES	CHANGE IN MASTERY PERCENTAGES							
	Non-mastery to mastery (*a*)		Mastery to non-mastery (*b*)		Non-mastery on both occasions (*c*)		Mastery on both occasions (*d*)	
	6th	1st Yr	6th	1st Yr	6th	1st Yr	6th	1st Yr
E. Decimals and Percentages								
The student can:								
1. Sequence decimals.	20	19	12	11	32	28	37	42
2. Convert a fraction to a decimal and vice versa.	23	17	9	11	17	19	51	53
3. Correctly position the decimal point.	25	17	10	11	45	49	20	24
4. Convert percentages to decimals and vice versa.	22	18	11	16	18	18	49	48†
5. Convert percentages to fractions and vice versa.	23	18	6	9	53	45	18	28
6. Calculate the percentage one whole number is of another.	25	18	9	10	49	40	17	32
7. Convert metric measures from one unit level to another.	24	19	15	14	33	32	28	35
F. Algebra								
The student can:								
1. Solve simple algebraic equations.	27	17	10	10	30	25	33	47
2. Convert a written problem into an open sentence.	25	19	13	13	24	20	38	48
3. Solve word problems algebraically.	22	18	11	10	37	26	29	47
4. Solve algebraic equations which call for two simple arithmetic operations.	16	19	6	11	70	54	8	16
5. Substitute values for the placeholder in simple algebraic expressions.	24	14	10	10	35	24	32	52

TABLE 2 (Contd)

OBJECTIVES	CHANGE IN MASTERY PERCENTAGES							
	Non-mastery to mastery (*a*)		Mastery to non mastery (*b*)		Non-mastery on both occasions (*c*)		Mastery on both occasions (*d*)	
	6th	1st Yr	6th	1st Yr	6th	1st Yr	6th	1st Yr
6 Select a correct number sentence from a set of number sentences containing inequalities	20	16	11	10	28	24	41	49
G Geometry								
The student can								
1 Identify common geometric forms	18	22	9	7	16	11	57	60
2 Define common geometric terms	23	17	12	10	41	45	24	28
3 Recognize facts about angles	23	21	16	14	28	27	32	38
4 Label the parts of a circle	25	14	10	12	13	13	52	61†
5 Calculate the perimeter of simple geometric shapes	21	17	13	11	26	20	40	52
6 Calculate the area of simple geometric shapes	21	13	7	7	57	56	15	23
H Charts and Graphs								
The student can								
1 Interpret charts and graphs	25	17	6	11	26	17	43	59
J Arithmetic Problems								
The student can								
1 Solve problems on speed	21	18	11	13	46	38	21	31
2 Solve problems on averages	22	13	6	7	47	48	25	32
3 Solve problems on value added tax	14	13	16	12	66	69	5	6†
4 Solve problems on interest rates	33	17	6	11	47	50	14	22
5 Solve problems on profit and loss	18	13	8	8	67	61	7	18
M	21 64	16 64	9 85	10 62	31 42	30 71	37 16	42 05
SD	4 42	3 47	2 76	2 41	17 48	16 28	18 67	17 49

† Objectives not registering net growth significant at the 01 level

TABLE 3

MEAN PERCENTAGE MASTERY LEVELS BY CONTENT SECTION

	Sixth class		First year post-primary	
	Beginning of year	End of year	Beginning of year	End of year
A Operations with whole numbers	69.75	78.00	69.75	75.00
B Whole number structure	53.70	64.40	58.20	65.60
C Fractional number structure	52.20	64.80	56.50	62.20
D Operations with fractions	46.60	59.10	47.90	52.70
E Decimals and percentages	41.90	54.40	49.10	55.60
F Algebra	40.30	52.70	53.80	60.30
G Geometry	48.20	58.50	54.00	61.20
H Charts and graphs	49.00	69.00	66.00	77.00
J Arithmetic problems	23.80	36.00	32.00	36.60

To obtain a measure of net growth for the data in Table 3, the beginning-of-year percentage is subtracted from the end-of-year percentage. All content areas registered significant net growth during the year; growth ranged from a low of 8.3% for operations with whole numbers (A) to a high of 20% for charts and graphs (H). The net growth for the remaining sections was similar, ranging from 10.3 to 12.6 percent. Order of mastery level was similar on the two occasions on which the test was administered; on both occasions, highest mastery was achieved for operations with whole numbers (A) and lowest for arithmetic problems (J).

First year post-primary

Table 2 also contains the results for the beginning and end-of-year testings of the first year post-primary sample. Across the 55 objectives, the average percentage of pupils who achieved mastery entering their first year in the post-primary school was 52.65 (total of means for columns *b* and *d*); at the end of the year the average percentage achieving mastery was 58.73 (total of means for columns *a* and *c*). Mean net growth over the 55 objectives for the year ($a - b$) was 6.0%, with a range from 1 to 15 percent.

The average percentage of pupils over the 55 objectives who showed mastery on both occasions was 42 1, while an average of 30 7% exhibited non mastery on both occasions As in sixth standard, movements between mastery and non mastery occurred on all objectives The mean percentage movement from non-mastery to mastery was 16 6% (with a range from 5 to 24%), while the mean percentage movement from mastery to non mastery was 10 6% (with a range from 5 to 16%)

Statistically significant growth was achieved on 40 objectives Of the 15 objectives which did not register significant growth, eight had been mastered by approximately 70% or more of pupils at the beginning of the year

Mean percentage mastery levels by major content area are set out in Table 3 All content areas registered significant growth during the year, growth ranged from a low of 4 6% for arithmetic problems (J) to a high of 11 0% for charts and graphs (H) While at the beginning of the year, the mean mastery level was over 50% for only three content areas (operations with whole numbers, whole number structure and fractional number structure), by the end of the year this level of mastery had been achieved for all content areas, with one exception (arithmetic problems) The order of mastery level for content areas, however, remained fairly similar from beginning to end of year, as was the case with sixth class pupils, on both test occasions, the highest mastery level was achieved for operations with whole numbers (A) and the lowest for arithmetic problems (J)

DISCUSSION

Before considering the growth of mathematical attainments during the final year of primary school and the first year of post-primary school, it is instructive to examine the level of performance of pupils at the beginning of these two periods Not surprisingly, the mean level of mastery across the 55 objectives of sixth class pupils (47%) was found to be lower than that of post primary pupils (53%) At both levels, there was great variation in mastery levels – from 12 to 90% for sixth class pupils and from 18 to 92% for first year post primary pupils However, for both groups, the majority of objectives had been mastered at the beginning of the year by between 30 and 70% of students, 39 objectives fell in this range for sixth class pupils and 41 for first year post primary pupils Furthermore, 36 of these objectives were common to both grade levels

In general, an equal or higher percentage of post primary than of primary pupils achieved mastery when they took the test for the first time For seven objectives, however, this was not the case, two of these involved operations with whole numbers, three involved operations with fractions, one involved identification of the least common multiple of two numbers and one, the solution of problems on value added tax. There was a wide range in the mastery level of these objectives.

There was considerable similarity between the two grade levels in the items for which high and low levels of mastery were attained. High levels of mastery – over 80% – were attained at both grades for the subtraction of numbers containing not more than five digits, the performance of simple whole number operations involving zero and the addition and subtraction of two fractions having the same denominator. The addition of a column of whole numbers (containing not more than five digits) was mastered by approximately 80% of students at both grade levels. At the other end of the mastery level, there was somewhat less consistency. However, the four objectives which were mastered by less than 20% of pupils at the primary level all had relatively low mastery levels (in the 20-30% band) in the post-primary sample also; division of a fraction by a whole number and vice versa had a level of mastery below 20% at both grades.

A consideration of objectives grouped into content areas, not surprisingly, reveals a similar pattern to that found for individual objectives. For both primary and post-primary pupils, the highest mastery level was achieved for operations on whole numbers. Relatively high mastery levels were also achieved on whole number structure and fractional number structure. The lowest level of mastery was achieved by both groups on arithmetical problems; operations with decimals and percentages also showed low mastery levels.

Within content areas, there was considerable variation in the mastery levels achieved for objectives. For example, for sixth standard pupils, mastery of objectives dealing with whole number structure ranged between 12% (identifying the distributive property) and 90% (performance of whole number operations involving zero). The range for post-primary pupils was from 21 to 92 percent; the high and low percentages at this level were associated with the same objectives as at sixth standard. Again, for operations with fractions, mastery levels varied between 17 and 81% for sixth class pupils and between 19 and 81% for post-primary pupils. The same objectives were involved at both levels; a low level of mastery was achieved for the division of a fraction by a whole number and vice versa, a high level for the addition and subtraction of two fractions with the same denominator. The range was not so great for other content areas. However, it is large enough in all areas to indicate the relative discreteness of students' skills in areas which are made up of objectives which involve content which seems conceptually similar.

An examination of movements of pupils between mastery and non mastery and vice versa indicates that overall loss is slightly higher for first year post primary pupils than for sixth standard pupils, while overall gain is slightly higher for sixth class pupils There is relative uniformity in the magnitude of both the loss and gain statistics both within and across classes

The magnitude of the loss statistic (which varied between 5 and 16% at both grade levels) may seem surprising At both grade levels, a sizeable percentage of pupils who had attained mastery at the beginning of the year had lost it by the end of the year At the sixth class level, for example, 16% of pupils who could recognize facts about angles and 15% of pupils who could multiply two fractions having the same denominator at the beginning of the year could not do so at the end Similarly, at the first year post primary level, 16% of pupils who converted percentages to fractions and vice versa at the beginning of the year did not do so at the end These percentages, while they no doubt to some extent reflect measurement error, also indicate that pupils lose as well as gain skills over the course of a year

Our data on net growth indicate that growth in sixth class was of the order of 12 percent Significant growth was achieved for all objectives with the exception of two The highest growth rates were achieved for arithmetical problems relating to interest rates, interpretation of charts and graphs, and a number of objectives involving fractions Growth on the objective of division of a number containing not more than three digits was also relatively high, since this operation would normally be considered to have been mastered by the end of fourth class, its initial low level of mastery must be regarded as surprising

Low growth, not surprisingly, occurred on items which had a high initial mastery level (e g , performing simple whole number operations involving zero) However, the solution of problems involving value added tax, which had a low initial mastery level, also showed little growth This invites the question does instruction in schools reflect the curriculum for this objective? It is also interesting to note that this objective is one of the few items in the test which involves the application of mathematical skills to a real world problem

Just as there had been considerable variation in initial mastery levels within content areas, there was also considerable variation in growth within content areas For example, net growth among objectives classified as operations with whole numbers ranged from 4 to 17 percent; among objectives classified as dealing with whole number structure, it ranged from 2 to 18 percent; and among arithmetical problems, from 2 to 27 percent. Variation was less in other content areas.

The pattern of results relating to net growth at the first year post-primary level are not greatly dissimilar to those at the primary level. For the post-primary sample, net growth was 6 percent. Significant growth was achieved on 40 of the 55 objectives. More than half of the objectives which did not show significant growth had been mastered by approximately 70 percent or more of pupils at the beginning of the year. Those items fell mainly in the categories dealing with operations with whole numbers and whole number structure. As at the sixth class level, insignificant growth was recorded for the objective, solution of problems on value added tax, which had a very low initial mastery level.

Growth on objectives within content areas showed considerable variation. Variation was greatest in the case of whole number structure (1 to 15%) and geometry (2 to 15%) and least in the case of algebra (4 to 8%) and arithmetic (1 to 6%).

In considering net growth, it is tempting to make comparisons between our samples of sixth class and first year post-primary pupils. Direct comparisons, however, present some difficulty. Firstly, it should be pointed out that the net measure of growth employed is based on the number of pupils who move from mastery to non-mastery as well as the number who move from non-mastery to mastery. If there are differences between objectives in the numbers of pupils who move in either direction, this might not always be discernible from the net growth statistic. Thus comparisons of net growth statistics could, in the case of some objectives, be misleading. In fact, as examination of Table 2 reveals, there was considerable similarity between objectives in the numbers of pupils moving from non-mastery to mastery and from mastery to non-mastery within each class level. Secondly, it should be borne in mind that since the number of pupils in the post-primary sample is smaller than that in the primary sample, a greater amount of growth will be required at the former level to achieve statistical significance. However, this problem is probably not as great as it might seem at first sight as the significant percentages of growth in the primary sample are, with two exceptions, all considerably greater than the amount required for statistical significance at the .01 level. (Only two objectives for which growth is significant exhibit a growth of 4% or less.) A final point to be borne in mind when comparing the performance of primary and post-primary pupils is that we are dealing with two independent samples Had the same sample of pupils been followed from primary to post primary school, a basis for comparing growth at the two levels would seem more obvious However, the similarity of the patterns of performance at the two levels suggests strongly that our samples represent performance of the population at each level and thus make comparison of the statistics obtained for our samples reasonable

With these reservations in mind, we may note that mean growth at the primary level (12%) exceeded mean growth at the post primary level (6%) This overall difference is of course also reflected in growth on individual objectives and on objectives grouped by content

Objectives on which differences – all in favour of the primary school pupils – were greatest were solution of problems on interest rates (21% difference), division of two fractions having the same denominator (15% difference), identification of prime numbers and composite numbers, stating a ratio as a fraction and labelling the parts of a circle (all showing a 12% difference) Growth at the post primary level was greater than growth at the primary level for five objectives, however, in most cases, the difference was marginal The objectives concerned were the subtraction of two numbers containing not more than five digits, identification of the distributive property, sequencing decimals in order of magnitude, identification of common geometric forms and recognition of facts about angles With one exception, these are objectives which one associates with secondary school curricula

Although the primary sample commenced the school year with a lower overall level of mastery (47%) than the post primary sample (53%), differential rates of growth at the two grades resulted in similarity in mastery levels at the end of the two grades In fact, the mean mastery levels at the end of sixth class and the end of first year post primary are almost identical (close to 59%) Here we may note that the post-primary sample did not begin at the level of mastery of pupils at the end of sixth class Beginning-of year mastery for first year post-primary pupils was 53% as compared to 59% for end-of-year mastery for sixth standard pupils Assuming equivalence of the samples, this represents a loss of 6% over the course of the summer holidays During their first year in the post-primary school, pupils manage to recover that loss, but no more

This is not to say that the post primary period is spent simply in recovering skills or knowledge that were lost In some cases, this seems so

For several objectives which exhibited a loss over the summer, pupils returned to approximately the same level of mastery as had been attained at the end of primary schooling (e.g., factoring two and three-digit numbers, identifying common factors between two numbers, stating a fraction in a number of equivalent forms, completing a ratio statement). In other cases, the extent of growth at the post-primary level is such that at the end of the year, the proportion of pupils exhibiting mastery is greater among post-primary than among primary pupils (e.g., positioning a number on the number line, identification of the distributive property, solution of word problems algebraically, solution of algebraic equations which call for two simple arithmetical operations; identification of common geometric forms). There are also objectives, however, for which mastery levels at the end of first year post-primary are lower than at the end of sixth standard. These occur particularly for objectives involving whole number structure (e.g., identification of prime numbers and composite numbers) and objectives involving fractions (addition and subtraction of fractions having different denominators, multiplication of fractions with the same denominator, division of fractions with the same and with different denominators).

These patterns, no doubt, reflect the different emphases in curricula and teaching at primary and post-primary levels. They may also be taken as an indication of attrition in skills which are not practised. It is not surprising, for example, that pupils should continue to show growth in algebra at the post-primary level, where this subject would be given particular attention. On the other hand, some objectives dealing with whole number structure show little improvement beyond the primary school level, perhaps because it is assumed that these have been adequately dealt with before the pupil enters post-primary school. It is surprising, perhaps, that a greater number of geometry objectives do not show a higher level of growth at the post-primary level; as with algebra, one would expect that this subject would receive particular attention at that level. The low level of growth for objectives dealing with arithmetical problems is also surprising. Pupils at the end of first year in post-primary school are little better in this area than pupils leaving primary school, who are not very well equipped at that stage.

In general, the findings of our study can be taken as providing some indication of the success and failure of mathematics instruction in fifth and sixth classes in the primary school and in first year in the post-primary school. While significant growth over a wide range of objectives was achieved at both levels, our findings indicate that many of the objectives of the curriculum for fifth and sixth classes in primary school are not being attained by the end of the primary school period Furthermore, if pupils begin post-primary schooling without having mastered the objectives of the primary school curriculum, it is unlikely that they will attain mastery of them during their first year in post-primary school

While for some objectives, a relatively high level of mastery was achieved by both primary and post primary samples, a large number of objectives had low initial mastery levels and also low levels of growth At the end of primary schooling, 16 objectives were mastered by less than half the pupils These objectives included three relating to fractions, three relating to decimals and percentages, and all five objectives dealing with arithmetical problems At the end of the first year in post-primary school the number of objectives mastered by less than half the pupils was 17, again, pupils seemed to experience greatest difficulty with objectives relating to fractions, decimals and percentages, and arithmetical problem solving

The finding that growth achieved in the first year of post-primary school was substantially less than growth in sixth class suggests that, although there is considerable overlap in the objectives of the curricula at both levels, there is less instructional emphasis on the common objectives in the post-primary school

Our findings, particularly at the primary school level, are interesting in the light of recent revisions of the curricula which, it has been suggested, have led to a shift in emphasis from the development of computational skills in pupils to the development of mathematical concepts and processes Had this shift in emphasis taken place, one might have expected less competence on objectives dealing with computation and more on objectives relating to concepts and problem solving Our results, however, indicate considerably higher competence in the computational area than in the area of mathematical concepts and problem solving Indeed, the low level of mastery attained in the area of problem solving was, perhaps, one of the most conspicuous of our findings

A number of suggestions for improving mathematical attainments in school may be made Firstly, detailed assessments of the attainments of individual pupils early in the school year should be of assistance to teachers in identifying pupils' skills and deficiencies at a stage when it is still possible to take remedial action Secondly, a programme of instructional and practice activities geared to the pupil's levels of attainments and aimed at developing mastery of objectives not yet mastered should be provided Some pupils may require more time than is at present available to master objectives (2, 4). Given time limitations in school, consideration should be given to the possibility that the existing curriculum is too extensive for all pupils to master. If this is so, then it would seem reasonable to attempt to identify socially useful skills and give these priority in instruction. If experience in the United States in minimal competency testing is to be taken as an indication (7) however, the identification and teaching of such skills is not going to be an easy task.

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