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OVER- AND UNDER-ACHIEVEMENT IN READING AND MATHEMATICS

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To test the hypothesis that over- and under-achievement in attainment will approximate to the normal distribution, standardized scores on group tests of reading and mathematics were regressed on non-verbal intelligence for each of four pupil samples, boys and girls at ages seven and ten. The hypothesis was confirmed. The further hypothesis that pupil over-achievement in mathematics and boys' under-achievement in reading will each exceed statistical expectation was confirmed for mathematics but not for reading.

Thorndike (6) has pointed out that, since correlations between intelligence and attainment are always less than perfect, the only satisfactory means of defining over- and under-achievement is through the regression equation. This avoids the assumption of the achievement ratio (4) that achievement age should exactly parallel mental age. It also, as Yule, Rutter, Berger, and Thompson (10) point out, avoids the sorts of conceptual defects in research noted by Crane (3) and Burt's (2) assumption that only in exceptional circumstances will children achieve at a level above their mental age.

A concomitant of these differing view-points is a dearth of empirical evidence on the distribution of over- and under-achievement among children of school age. Recently, however, Yule *et al* (10) have used the regression equation to provide estimates of over- and under-achievement for five regional age-groups in England. They found that over- and under-achievement in reading followed the expected normal distribution except for the extremes of the ranges. Under-achievers were more, and over-achievers were less common than expected. They point out, however, that estimates for the extremes were depleted by floor and ceiling effects, those for over-achievement being most affected by lack of ceiling on the reading tests.

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The present study is concerned with the distribution of over and under achievement in reading and mathematics among pupils of primary school age in Northern Ireland It takes account of a series of investigations, reviewed by Wilson (9), which had shown a consistent pattern of superior arithmetic and inferior English performance by Northern Ireland pupils on tests standardized elsewhere in the United Kingdom Moreover, a recent comparison between the reading standards of eleven-year olds in Northern Ireland and England had shown a greater incidence of poor readers among the Northern Ireland boys (8) Differences for girls at this age were negligible

The study is therefore designed to test the hypothesis that reading and mathematics, regressed on non-verbal intelligence, will each be normally distributed, except that under achievement in reading and over achievement in mathematics will be greater than expected, and that under-achievement in reading will be greater for boys than girls

METHOD

The study follows on from an investigation conducted by the Northern Ireland Council for Educational Research (7) in which test data were available for 4,554 seven- and ten year old pupils in 115 schools

These schools represented a one in ten probability sample of all Northern Ireland primary schools The second stage pupil samples included all seven- and ten year old pupils in the first stage sample of schools

Test scores on group tests of reading comprehension and mathematics were available for both age groups In addition, the seven year olds completed a group test of 'picture intelligence' and the ten-year olds completed group tests of verbal and non verbal ability For the present study the non verbal test was chosen in preference to the verbal test as a measure of scholastic aptitude for ten year olds The choice was made on the grounds that a group test of verbal ability is so dependent on an adequate level of reading competence as to confuse the aptitude attainment distinction for the less able reader

All but one of the six tests used in the present study were standardized and normalized on the age samples for sexes combined The published test norms were used to derive standardized scores for the Moray House Picture Test The test score characteristics and intercorrelations for each of four samples, boys and girls at each age level, are presented in Table 1

TABLE 1

SAMPLE AND TEST CHARACTERISTICS

Samples	Tests	Mean	SD	Intercorrelations	
7-year-old boys	M.H. Picture Intelligence	93.5	15.8		
n=1267	Reading Test NS45	98.9	15.4	0.683	
	Mathematics A1	99.1	13.6	0.740	0.667
7-year-old girls	M.H. Picture Intelligence	96.9	15.6		
n=1103	Reading Test NS45	103.7	15.3	0.704	
	Mathematics A1	100.6	13.5	0.703	0.679
10-year-old boys	Non-verbal 5	99.5	15.0		
n=1178	Primary Reading 2	98.3	15.0	0.720	
	Mathematics C3	99.6	15.4	0.803	0.822
10-year-old girls	Non-verbal 5	100.8	14.3		
n=1006	Primary Reading 2	101.8	13.7	0.699	
	Mathematics C3	101.2	14.1	0.776	0.795

At age seven the test means obtained by girls are all superior beyond the one per cent level of significance to those obtained by boys, with the means for reading comprehension showing the largest difference (5, formula 7 2) At age ten the girls' mean score on reading is superior beyond the one per cent level The girls' mean scores on non-verbal intelligence and mathematics are superior beyond the five per cent level None of the sex-differences for sample standard deviations is significant at age seven, but at age ten on the tests of attainment those for boys are significantly greater than those for girls beyond the one per cent level (5, formula 6 11)

For seven year old boys, ten year old boys and ten year old girls the correlations between non verbal intelligence and mathematics are significantly greater beyond the one per cent level than those between non-verbal intelligence and reading (5, formula 10 7) For seven year old girls the difference is clearly not significant

RESULTS

Regression equations were used to compute predicted reading and mathematics quotients for each of the four groups The actual attainment quotients *less* the predicted attainment quotients provided a set of discrepancy distributions, each with a mean of zero and a standard deviation which is also known as *the standard error of prediction*

The observed pupil Ns and percentages for each discrepancy distribution are presented for standard error intervals in Table 2 Pupil Ns, as expected on the basis of the normal distribution, are also shown for each interval The percentage basis on which the expected Ns are calculated is shown as the final row of the table

To test the hypothesis that the attainment discrepancy distributions as shown did not depart significantly from the normal distribution a series of Chi square tests for goodness of fit (5, pp 231-235) were applied to observed and expected Ns in Table 2 The values of Chi-square are included in the table The appropriate degrees of freedom are given as $k - 3$

Two distributions, reading discrepancy for seven year old boys and mathematics discrepancy for seven year old girls, show departures which are significant beyond the five per cent level The major departure in reading for seven year old boys is at minus one to minus two standard errors, where the number of boys is greater than expected The major departure for seven-

TABLE 2

OBSERVED AND EXPECTED DISTRIBUTIONS FOR READING AND MATHEMATICS DISCREPANCIES

Samples	Attainments	< -2SE		-2SE to -1SE		-1SE to +1SE		+1SE to +2SE		>2SE		χ^2 (df =2)
		n	%	n	%	n	%	n	%	n	%	
7-year old boys n=1267	Reading Obs.	24	1.89	199	15.71	841	66.38	176	13.89	27	2.13	5.99*
	Maths Obs.	24	1.89	167	13.18	874	68.98	170	13.42	32	2.53	1.43
	Expected	29		172		865		172		29		
7-year old girls n=1103	Reading Obs.	31	2.81	141	12.78	755	68.45	159	14.42	17	1.54	5.09
	Maths Obs.	18	1.63	164	14.87	762	69.08	127	11.51	32	2.90	8.86*
	Expected	25		150		753		150		25		
10-year old boys n=1178	Reading Obs.	28	2.38	155	13.16	816	69.27	149	12.65	30	2.55	1.46
	Maths Obs.	21	1.78	170	14.43	805	68.34	147	12.48	35	2.97	4.39
	Expected	27		160		804		160		27		
10-year old girls n=1006	Reading Obs.	24	2.39	139	13.82	687	68.29	138	13.72	18	1.79	1.17
	Maths Obs.	19	1.89	131	13.02	692	68.79	137	13.62	27	2.68	1.71
	Expected	23		137		686		137		23		
All groups n=4554	Reading Obs.	107	2.35	634	13.92	3099	68.05	622	13.66	92	2.02	1.88
	Maths Obs.	82	1.80	632	13.88	3133	68.80	581	12.76	126	2.77	12.12**
	Expected	104		619		3108		619		104		
%age basis of expectation			2.28		13.59		68.26		13.59		2.28	

* $p<0.05$ ** $p<0.01$

year old girls is at plus one to plus two standard errors, where the number of girls is less than expected Since none of the remaining distributions shows a significant departure from expectation, it may be concluded that the main hypothesis is substantiated

However, the hypothesized normality of attainment discrepancy distributions was conditional on the further expectation that boys in particular would under achieve in reading, whereas all age groups would over achieve in mathematics If, following Angoff (1) and Yule *et al* (10), over and under achievement are defined as plus and minus two standard errors of prediction, there is clearly no evidence of excessive over or under achievement in reading for any age group or for age groups combined For combined age groups, departure from normality in mathematics is significant at the one per cent level The major departures are at the extremes of the range, with less than expected under achievement and more than expected over achievement, as hypothesised

Floor and ceiling effects

The minimum IQ required for an attainment quotient of 70 to be two standard errors below prediction and the maximum IQ for an attainment quotient of 140 to be two standard errors above prediction were calculated for each criterion, as follows

let X_1 = minimum IQ
let X_2 = maximum IQ
let B = the unstandardized regression coefficient
let A = the regression constant

Then for under achievement,

$$70+2SE = BX_1+A$$

so that

$$X_1 = (70+2SE-A)/B,$$

and for over achievement,

$$140-2SE = BX_2+A$$

so that

$$X_2 = (140-2SE-A)/B$$

The minimum and maximum IQ values for under and over achievement are shown in Table 3 The table also shows the corresponding percentage estimates of numbers of pupils among whom over and under achievement went undetected because of floor and ceiling effects on the attainment tests

TABLE 3

TEST LIMITS FOR OVER- AND UNDER-ACHIEVEMENT

Sample	Criterion	Under-achievement Min. IQ	Under-achievement %age excluded	Over-achievement Max. IQ	Over-achievement %age excluded
7-year-old boys	Reading	84	27	122	4
	Maths	77	15	129	1
7-year-old girls	Reading	79	13	118	9
	Maths	82	17	134	1
10-year-old boys	Reading	89	24	129	2
	Maths	86	18	126	4
10-year-old girls	Reading	82	10	129	2
	Maths	83	11	128	3

Test ceiling effects are so slight as to have had little effect on estimates for over-achievement. Floor effect is much more considerable, particularly in reading for boys at both age levels, where roughly a quarter of each age group is excluded from the estimates of reading under-achievement in Table 2. If the estimates are weighted to take account, *pro rata,* of the artefactually excluded quartiles, the estimates are increased from 1.9 and 2.4% to 2.5 and 3.1% at ages seven and ten.

For girls, the weighted estimates are increased from 2.8 and 2.4% to 3.2 and 2.6% at ages seven and ten. None of the revised estimates indicates an unusual degree of under-achievement when compared with an expected population percentage of 2.3.

CONCLUSION

Over- and under-achievement are seen to occur with roughly equal frequency in both reading and mathematics at the primary stage of education. The evidence illustrates clearly that lack of perfect correlation between intelligence and attainment will ensure that as many children will achieve above as below prediction on the basis of intelligence, as argued by Thorndike (6) and demonstrated empirically by Yule *et al* (10). The assumption that mental age imposes a fixed limit on attainment and that therefore over-achievement is both unnatural and rare is refuted.

The further hypothesis that a traditional pattern of superior arithmetical and inferior English performance would be reflected in over achievement in mathematics is confirmed overall, though not within any of the four groups It is interesting to find the phenomenon detectable on tests designed to sample mathematical understanding, since the studies on which the hypothesis is based had detected arithmetical superiority most strongly on tests of mechanical arithmetic This had led to the criticism that an excessive emphasis was being placed on arithmetical rote practice in Northern Ireland schools to the detriment of mathematical understanding

The lack of any substantial confirmation of a greater than-expected incidence of under achievement in reading, particularly for boys, is surprising Yule *et al* (10) found extreme under achievement occurring at a rate appreciably above expectation Even when test artefacts in depressing the estimates are allowed for, the present study's estimates only marginally exceed three per cent for two groups, one of which is seven year old girls Since incidence of under achievement in reading is not excessively above expectation for either sex, the sex differences in reading performance, noted in Table 1, are clearly not attributable to differential under-achievement in reading Instead, the evidence may point to a more generalized sex difference in test taking aptitude at these ages, as reflected on all of the group tests

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