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THE JUNIOR CYCLE CURRICULUM AND THE PISA MATHEMATICS FRAMEWORK

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The results of an analysis of the intended, implemented, and achieved Irish junior cycle mathematics curriculum using the Programme for International Student Assessment (PISA) mathematics framework, assessment instruments, and results, as benchmarks are reported. First, the PISA mathematics framework, its theoretical roots, development, and structure, are described. The extent to which the intended junior cycle mathematics curriculum measures up to the PISA framework on a number of key aspects (including goals, principles, syllabuses) is considered. Substantial differences in a number of these aspects emerge. The way in which the mathematics curriculum is implemented in Irish schools was judged to be at variance in many ways with methods that are implied by the PISA framework. In considering the achieved curriculum, Junior Certificate examination (JCE) papers were evaluated with reference to the PISA assessment framework. Examination papers were found to differ from PISA, and by examination level, in the percentage of items assessing overarching ideas and competency clusters and in presenting items in context. Correlations between performance on the JCE and PISA were high, but not as high as between performance on PISA tests of mathematics, reading, and science.

Mathematics is a key subject in school curricula. Traditionally it is viewed as a subject of study in its own right and, as such, has a number of facets. It can be approached as the cumulative study of abstract systems, the study of patterns, or the study of a rigorous and abstract language for communicating mathematical ideas, processes, and proofs, some of which find important applications in science and engineering. Alternatively, mathematics can be viewed more as a tool and medium for facilitating study in many academic subjects, particularly the physical and social sciences, for solving problems in our personal, educational, and occupational lives, and generally for studying and making sense of the world around us. Most of the technological tools of today require some level of mathematical knowledge and reasoning to understand how they work and how best to use them in solving problems. The traditional subject-centred view of mathematics has dominated most school mathematics curricula for the past hundred years or so despite the dramatic economic, social, scientific, and technological changes that have taken place in society, changes that require a greater emphasis on the use of mathematics as a tool of human activity and means of communication and less as an isolated area of study.

The OECD Programme for International Student Assessment (PISA) takes an instrumental view of mathematics and was set up to look at how well students approaching the end of their compulsory period of schooling could use mathematics to deal with problem situations in the world around them, rather than how much they had learned of their school mathematics curricula. This trend represents a move away from a more elitist and traditional view of mathematics as something at which only the more able few can be successful, to one in which mathematics is seen as a subject at which everyone needs to become proficient to some degree. The earlier view led to a situation where mathematics curricula contained a lot of content that was learned superficially by most students. This approach resulted in many students acquiring a range of rather disconnected and decontextualized procedural and conceptual knowledge that they tended to associate mainly with mathematics class and not much with the world outside the classroom. The PISA mathematics framework was a conscious effort to move comparative international assessment away from curriculum-dominated assessment and represented a substantial innovation in mathematics education and in the approach to assessing outcomes of mathematics teaching and learning across countries. It was quite different in its goals and assessment tasks from the more curriculum-orientated approach of international comparative surveys such as TIMSS (Beaton, Mullis, Martin, Gonzalez, Smith, & Kelly, 1996)) and IAEP (Lapointe, Mead, & Phillips, 1989).

This paper provides analyses of the intended, implemented, and achieved Irish junior cycle mathematics curriculum using the PISA mathematics framework and philosophy, assessment instruments, and results, as benchmarks. Before describing the results of these analyses, the rationale and development of the PISA mathematics framework are outlined. Following that, the intended junior cycle mathematics curriculum is compared with the PISA framework by examining the extent to which they correspond on a number of key aspects (including goals and principles). Then, available evidence on how the curriculum is implemented in schools and the extent to which teaching methods are compatible with methods implied by the PISA framework are considered. Finally, the achieved curriculum is considered from two perspectives: how items on the Junior Certificate examination map on to the PISA assessment framework, and the extent to which students' performance on the Junior Certificate examination matches their performance on PISA. Implications of the results of the analyses for the junior cycle curriculum are drawn and followed by some tentative recommendations.

THE PISA MATHEMATICS FRAMEWORK

The PISA mathematics survey focuses on assessing how well students can use the mathematics they have learned in realistic situations (referred to as *mathematical literacy*) (OECD, 2003). While previous international studies such as IAEP and TIMSS had selected items that were common to the mathematics curricula of participating countries, PISA did not consider country curricula in the selection of items. The move away from the curriculum-based assessment of earlier international studies generated considerable debate among mathematics education researchers. Some countries' mathematics curricula were close to the PISA framework in terms of the mathematics taught (e.g., Netherlands, Australia), while others were quite distant (e.g., Ireland, Portugal). Some mathematics educators and measurement specialists felt that students taking the tests should not be asked to respond to unfamiliar tasks to which they had no previous exposure in school. On the other hand, there were those who felt that the mathematics curricula in many countries had been relativity unchanged since the 'modern mathematics' movement of the 1960s and needed to be revised to better serve the students of today's world and therefore could include tasks requiring problem-solving in situations not necessarily familiar to students. International comparative studies such as PISA highlight the need for change and reflect a more general move towards more broad-based and more valid assessment (Kulm, 1990). This move raises the more general issue of how well these studies can achieve their goals in light of present testing constraints which include time limitations, students' perceptions of the importance of the tests, cost constraints, cultural bias, and the problems of media use of international league tables.

Theoretical Roots

The underpinning philosophy and principles of the PISA mathematics framework and tests can be related to three theoretical viewpoints: (i) socio-cultural literacy; (ii) situated (problem-based) learning; and (iii) authentic assessment (Romberg, 2000). It is also seen as having strong links to an increasingly popular trend in mathematics education referred to as the Realistic Mathematics Education movement which originated in The Netherlands under the influence of Hans Freudenthal (Oldham, 2002).

In the context of socio-cultural literacy theory, which is a general theory of literacy (Gee, 1998; Vygotsky, 1978; Wertsch & Rupert, 1993), mathematics is seen as a language which has 'design' or technical features (e.g., concepts, rules, procedures), and to be *mathematically literate* means being able to use some of these features for different social functions, including, but not confined to, ‘everyday mathematics’. For example, knowing how to divide two numbers means knowing a design procedure but does not mean that one understands its structure or that one can use it to find, for example, the number of coaches needed for a school tour. This view parallels the view that learning the technical features (e.g., grammar) of a foreign language may not be sufficient to enable a person to use it effectively in the social and cultural milieu of the country where the language is used. PISA is concerned with how one becomes mathematically literate and, therefore, how to assess levels of mathematical literacy.

The second theoretical basis, situated learning theory, suggests that learning in any conceptual domain is a result of experiences – activities designed to facilitate movement from informal ideas to more formal and abstract ways of representing and reasoning in the domain of mathematics (Greeno, Collins, & Resnick, 1996; Greeno, Pearson, & Schoenfeld, 1997). It assumes that learning in a domain should begin with a situated problem that makes sense to the learner. The learner extracts the mathematics from its context and can then use it to solve the problem or subsequently to solve similar or more complex problems. This process is referred to as ‘horizontal’ mathematization. The extracted mathematics can also be analysed, refined, and developed without reference to any context, a process which is referred to as ‘vertical’ mathematization, and is the mainstay of traditional mathematics curricula. The PISA framework document focuses on horizontal mathematization and describes it as a five-stage process:

(i) starting with a problem situated in reality;
(ii) organizing it according to mathematical concepts;
(iii) gradually trimming away the reality through processes which promote the mathematical features of the situation and transform the real problem into a mathematical problem that faithfully represents the situation;
(iv) solving the mathematical problem; and
(v) making sense of the mathematical solution in terms of the real situation (OECD, 2003).

Acceptance of the socio-cultural view of mathematics as a form of literacy implies acceptance of the importance of situation and context in mathematics learning. The PISA framework reflects these views by the presence of a situation or context dimension in the framework as well as a content dimension which classifies mathematics phenomena in terms of broad general themes called ‘overarching ideas’.

A third basis of the PISA mathematics framework is ‘authentic assessment’ which (as it applies to mathematics education) represents, among other things, an attempt to expand assessment tasks to include more complex and situated problems. It is not considered enough to assess basic mathematical concepts, procedures, and simple applications which are easy to assess; there is also a need to assess how well students can use the technical features of mathematics to solve increasingly complex problems. Authentic assessment also means that process should be assessed as well as content. PISA does this by including a process dimension which classifies the processes used in a problem into clusters representing different levels of complexity and competency.

Development of the Framework

The work of developing the PISA assessment framework and tests for mathematics commenced in March 1998 with the first meeting of the Mathematics Expert Group (MEG) in Providence, Rhode Island, USA under the chairmanship of Jan deLange of the Freudenthal Institute in The Netherlands. The group included leading mathematics educators from ten countries, including Ireland, and held five subsequent working meetings of from 2 to 5 days in The Netherlands during the first cycle of PISA. The initial framework and test emerged following a review and discussion of existing national and international assessment frameworks and trends in mathematics education in general. A forum was also set up to enable mathematics educators from participating countries to meet with the expert group and express their views on the framework. Following the PISA 2000 assessment, the group (with some changes in membership) met six times (including a meeting in Dublin in July 2001) to further refine the framework and tests in preparation for PISA 2003 in which mathematics was the major focus. The novelty of the framework led to considerable resistance on the part of some country representatives, but it eventually received general approval of the participating countries.

Structure of the Framework[1]

While international surveys of school mathematics such as IAEP and TIMSS used a two-dimensional structure for the design of assessments, one dimension focusing on content (e.g., number, algebra, geometry, statistics) and the other on cognitive behaviour (e.g., recall of facts or definitions, carry out procedures, interpret data or patterns, solve problems), the PISA framework has three dimensions: *context*, *content*, and *competency*.

[1] A description of the PISA 2000 version of the framework is provided by Oldham (2002).

Context. Mathematical problem situations and contexts are categorized as (i) *personal*; (ii) *social/occupational*; (iii) *public*; and (iv) *scientific*, with varying kinds and levels of mathematics arising in these different types of situations and contexts. In the first PISA cycle of assessment (PISA 2000), a fifth category was used, *intra-mathematical*, explicitly accommodating situations in which the context for the question was provided by mathematics itself. However, in PISA 2003, this category was treated as part of the scientific domain.

Content. The mathematical content is described in terms of four categories that encompass the kinds of problems arising through interaction with everyday phenomena. In PISA, these are called the four 'overarching ideas': (i) *quantity*; (ii) *space and shape*; (iii) *change and relationships*; and (iv) *uncertainty*.

Competency. The basic cognitive processes identified as being involved in the more general process of mathematization include the following: reasoning; argumentation; communication; modelling; problem posing and solving; representation; using symbolic, formal and technical language and operations; and use of aids and tools. A mathematical task may involve one or more of these processes at various levels of complexity. Following on from this, the competency dimension is divided into three clusters of processes at different levels of complexity to reflect the varying cognitive demands of mathematical tasks. The clusters are: (i) *reproduction* – processes involved in performing calculations, solving equations, reproducing memorized facts or 'solving' well-known routine problems; (ii) *connections* – processes involved in integrating information, making connections within and across mathematical domains, or solving problems using familiar procedures in context; (iii) *reflection* – processes involved in recognizing and extracting the mathematics in problem situations, using that mathematics to solve problems, analysing and developing models and strategies, or making mathematical arguments and generalizations.

This framework underpins the PISA mathematics tests. The PISA tests, based on the framework, consist of a number of items, each of which constitutes a *task* for students. In the 2003 tests, there were 85 such tasks, not all of which were answered by each student. A task (usually situated in a realistic context) could involve several mathematical topics and several cognitive process skills. Sets of tasks (two to six items) are frequently based on the same stimulus material, which may include drawings, diagrams, and photographs as well as text[2].

[2] Examples are given in OECD (2003) and Oldham (2002).

Assessing mathematical process is not easy since exemplary problems that have the potential to promote the development of more complex mathematical processes in students can be reduced to a set of routinized skills which students can learn procedurally, thus bypassing the opportunity to engage in complex mathematical thinking and develop greater understanding. For example, in England, where the assessment of mathematical investigations was introduced to the GCSE examination in the 1980s, in many classrooms this was taught as an additional piece of content, with students given a set of procedures to follow (Smithers, 2004). One of the challenges facing teachers in implementing new curricula is how to integrate mathematical process skills with content topics/objectives.

Development of the Assessment Tasks

The PISA framework specified the direction to be taken in the assessments. The items used in the 2003 study were selected from a larger pool of items that had been tested in a field trial conducted in all countries one year prior to the main study. The PISA 2003 test was basically a paper-and-pencil test and was made up of multiple-choice, short-answer, and extended-response items. Extended-response items required more extensive writing, or showing a calculation, and frequently included some explanation or justification. Pencils, erasers, rulers, and in some cases calculators, were provided for students in the light of standard national practice. No items required a calculator, but some items involved solution steps for which the use of a calculator could facilitate computation. Irish students were provided with calculators as they are available to them in their normal classroom work and in the Junior Certificate examination.

A comprehensive set of guidelines was developed to encourage the submission of items from as many participating countries as possible. The 500 or so items submitted by 15 countries varied greatly, reflecting to some extent differing interpretations of the framework. Items were also prepared by the mathematics expert group. All items were subjected to a rigorous selection procedure which included the following steps: initial editing; item panelling by the mathematics expert group; talk aloud try-outs with individual students or small groups of students; international item panelling; and pilot testing in field trials (see OECD, 2005). Given the innovative nature of many of the items, there was considerable debate by the international mathematics expert group, by national representatives at the Mathematics Forum, and by national PISA committees about the suitability of items and how they related to the mathematics framework and to national curricula.

THE INTENDED CURRICULUM: THE JUNIOR CYCLE MATHEMATICS CURRICULUM AND THE PISA FRAMEWORK

Substantial variation in content, methodology, and assessment of mathematics curricula exists across countries and school systems, making it difficult to compare the curricula of countries and systems. Earlier international assessments such as SIMSS and TIMSS 1995, 1999, 2003 (Mullis, Martin, Gonzalez, & Chrostowski, 2004) identified common curricular content across countries and then used this content to develop an assessment framework for generating test items.

One way to seek to relate the PISA framework to a country's mathematics curriculum is to analyse each PISA item separately in terms of its content/cognitive behaviour dimensions, list the specific content and cognitive process/behaviour involved, and then compare the listings with those of the national curriculum or assessment framework (see Appendix for a list of content in the 2003 PISA pilot items; cognitive processes are as listed in the previous section). A drawback of this approach is that it decomposes the mathematics into a hierarchy of isolated skills and concepts without reference to the context in which they occur and, consequently, fails to consider relationships and connections among them which can occur in realistic situations and problems (Romberg & Zarinnia, 1987).

A preferable method is to analyse each item in terms of the three dimensions of the PISA framework and list the competency cluster needed to produce the appropriate response and then compare the resulting matrix with the country's curriculum framework. The first part of this work has been done by the PISA team with the 2003 items (see OECD, 2004, pp. 51-85 and OECD, 2005, chapter 16 for examples) and can be modified and used for particular national or local situations. Such an approach can enable a review team to design a curriculum which addresses mathematical literacy to a greater degree, with appropriate emphasis on tasks and investigations involving reproduction, connections, and reflection competencies or competency clusters. It should be pointed out that the PISA items themselves represent a rather restricted set of tasks in terms of scope of content, competencies, competency clusters, and situations reflected in the framework. This is due mainly to the constraints of international assessments arising from the limited time available for testing, the novelty (and hence difficulty) of the PISA items and item formats, and the exclusion of items due to perceived cultural bias. However, the PISA framework provides a broader view than just the scope of the items of what kind of tasks should be part of a mathematics curriculum and assessment programme that is oriented towards developing mathematical literacy. Using the framework, its philosophy, and its assessment instruments, it is possible to make a fairly comprehensive comparison with national curricula and, in the case of this paper, with the Irish post-primary junior cycle mathematics curriculum.

In considering a comparison of the PISA mathematics framework and assessment instruments with the junior cycle mathematics curriculum, the following questions are considered:

- How do the goals of the Irish mathematics curriculum compare with the PISA goal of mathematical literacy?
- How do the PISA framework and items relate to the Irish mathematics syllabus?
- How familiar are the PISA items to Irish students?

Answers to these questions can help explain Irish performance on the PISA mathematics tests and place the Irish mathematics curriculum in the context of international practice in mathematics education. In doing so the Junior Certificate mathematics assessment framework is also made clear.[3] The information and data used to answer the questions are drawn, in the main, from a number of sources. These include the PISA 2003 assessment framework (OECD, 2003); the PISA 2003 technical report (OECD, 2005); reports on PISA results for Ireland (Cosgrove, Oldham, & Close, 2005; Cosgrove, Shiel, Oldham, & Sofroniou, 2004; Cosgrove, Shiel, Sofroniou, Zastruzki, & Shortt, 2005); DES and NCCA mathematics curriculum and review documents for junior cycle mathematics (DES/NCCA, 1999, 2000, 2002; NCCA, 2005, 2006); the PISA 2003 test-curriculum rating project (Cosgrove, Shiel et al., 2005); a study of Irish mathematics classrooms (Lyons, Lynch, Close, Sheerin, & Boland, 2003); and an analysis of the Junior Certificate examination papers using the PISA framework (Close & Oldham, 2005).

Goals and Principles

As well as personal development, the main goals of the junior cycle mathematics curriculum focus on providing students with the mathematical knowledge, skills, and understanding needed for continuing their education and eventually the mathematics needed for life and work (DES/NCCA, 2000, 2002). The goal of the PISA framework is mathematical literacy – the ability of students to use the mathematics they have acquired in and out of school to solve mainly realistic problems. These statements might suggest that mathematical literacy in

[3] More details of the Junior Certificate examination framework are provided by Close & Oldham (2005).

the Irish curriculum is secondary to preparing students for further study of mathematics, that is, the senior cycle mathematics curriculum. For the vast majority of students, this is about the development of technical concepts and skills. The main goals of the junior cycle programme are divided into ten sub-goals or assessment objectives (objectives A to J in the official documents). The first four objectives relating to recall of facts (A), instrumental understanding (B), relational understanding (C), and application of mathematical knowledge (D) are the main focus of junior cycle work. Most of the ten objectives relate well to the PISA framework but six of them are not currently assessed in the Junior Certificate examination in a formal or substantive way. These are objectives relating to mathematics in unfamiliar contexts (E), creativity in mathematics (F), motor skills in mathematics (G), communicating mathematics (H), appreciation of mathematics (I), and history of mathematics (J). In the case of communicating mathematics, the present practice in Junior Certificate examinations of asking students to show their work for selected questions or question parts is done for the purposes of awarding partial credit as well as assessing the qualitative aspects of students' ability to express their mathematical thinking and understanding.

Cosgrove, Oldham and Close (2005) referred to the divergence between the goals and objectives of PISA and junior cycle mathematics, pointing to the fact that objectives that are assessed in the Junior Certificate examination (A, B, C, and D above) are likely to be accorded higher priority by teachers than objectives that are not assessed (E, F, G, H, I, and J above), a view supported by Irish mathematics teachers' responses to a questionnaire regarding levels of emphasis given to aspects of the Junior Certificate syllabus (Cosgrove et al., 2004).

As well as general principles relating to (i) breadth and balance, (ii) depth, and (iii) relevance, which apply to the junior cycle curriculum in general (DES/NCCA, 2000), there are three additional principles set out for the mathematics curriculum in particular (DES/NCCA, 2002). These recommend that (a) mathematics syllabuses should provide continuation from and development of the primary school curriculum; (b) that syllabuses should be implementable; and (c) that the mathematics they contain should be sound, important and interesting. With regard to (a), an analysis of the mathematics curriculum for senior classes in primary school and textbooks for first and second year classes in post-primary school reveals serious discontinuities in the content of the two syllabuses which, logically, would detrimentally affect the 'implementability' of the curriculum. This problem has been highlighted in reports about transition from primary to post-primary school (e.g., Smyth, McCoy, & Darmody, 2004), but little has been done to improve continuity .

Junior Cycle Mathematics Syllabuses and PISA Mathematics Framework

The PISA Irish report *Education for Life* compared the three junior cycle mathematics syllabuses, Higher, Ordinary, and Foundation levels, in terms of content strand and sub-topics (Cosgrove, Shiel et al., 2005, Table 6.1). A summary of the information is presented in Table 1.

Table 1
Numbers of Sub-Topics as Approximate Weightings of the Content Topics in the Junior Cycle Syllabus

	Number of Sub-Topics per Syllabus Level		
Content Area	Higher	Ordinary	Foundation
1. Sets	9	8	4
2. Number Systems	16	16	7
3, Appl. Arith. and Measure	11	11	9
4. Functions and Graphs	10	10	4
5. Algebra	10	10	4
6. Geometry	22*	16*	10*
7. Trigonometry	4	3	0
8. Statistics	7	4	4
Total	88	75	38

* Due to the need to give more detail re the Geometry section of the syllabus this number can be considered to reflect communicative convenience more than mathematical weighting of Geometry in the syllabus.

The junior cycle mathematics curriculum is divided into three levels for the purpose of streaming for teaching and examination. The Higher level syllabus was followed in 2003 by about 40% of students; the Ordinary level by a little over 50%; and the Foundation level by less than 10% of students. Table 2 provides a list of content subtopics which are on the Higher level but not on the Ordinary level syllabus. A more detailed comparison of the content of the three levels can be found in the Irish PISA report (Cosgrove, Shiel et al., 2005). As can be seen from Tables 1 and 2, there is little difference between the Higher and Ordinary level courses with just a small number of topics on the Higher level which are not on the Ordinary level. The Foundation level syllabus, on the other hand, is much more limited in content with a total of only 38 topics.

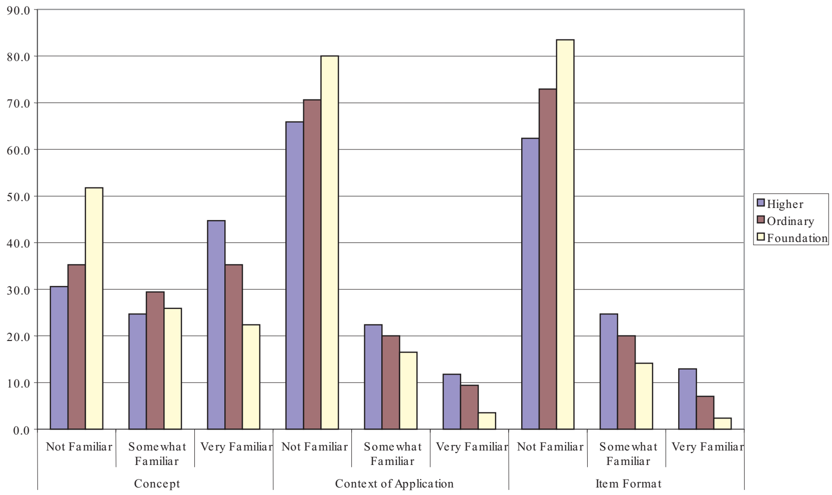
Table 2
Content Subtopics which are on the Junior Cycle Higher Level Syllabus but not on the Ordinary Level Syllabus

Content Area	Sub-Topics on the Higher Level Syllabus but not on the Ordinary Level Syllabus
Sets	(i) Distributive property
Number Systems	(i) Irrational numbers. (ii) Negative and fractional indices
Appl. Arithmetic and Measure	Same on both
Functions and Graphs	(i) Maxima and minima of quadratic functions from graphs (ii) Solution of quadratic inequalities from graphs
Algebra	(i) Division of expressions
Geometry	(i) Exterior angle of triangle theorem (ii) Angle at centre of circle theorem
Trigonometry	(i) Trigonometric ratios for 0 to 360 (ii) Surd forms of simple trig. ratios (iii) Use of sine rule (iv) Use of trig formulae to find area
Statistics	(i) Histograms: drawing and interpreting (ii) Cumulative frequency ogive and interquartile range (iii) Grouped frequency data

Familiarity of Irish Students with PISA Items

As part of the analysis of PISA 2003 in Ireland, the familiarity of Irish students with the concepts, contexts, and formats of the PISA items was assessed by a team of experienced mathematics teachers (Cosgrove, Shiel et al., 2005). The results indicated that, firstly, about a third of the concepts in PISA items were unfamiliar to Higher and Ordinary level students, and about half were unfamiliar to Foundation level students. Secondly, about two-thirds of the contexts or applications of the items were unfamiliar to Higher and Ordinary level students, and about four-fifths were unfamiliar to Foundation level students. Thirdly, about two-thirds of the formats of PISA items were unfamiliar to Higher and Ordinary level students, and about four-fifths were unfamiliar to Foundation level students. Finally, items in the *uncertainty* content category were least familiar, but students performed well on them. Items in the *reflection* cluster of competencies were least familiar (Figure 1).

Figure 1
Percentages of PISA 2003 Test Items with which Junior Cycle Students were Judged to be Familiar, by Junior Cycle Level



As part of the rating project, the PISA items were mapped on to the junior cycle syllabus content areas. The results are summarized in Table 3. Around 30% of the mathematical concepts of the items do not fall into any of the junior cycle content areas, including concepts in areas such as probability. None of the PISA items falls into the junior cycle areas of Geometry and Trigonometry, whereas, the largest proportion of items, about one-third, fall into the category of Applied Arithmetic and Measure.

The data and analyses presented in this section highlight substantial differences between the content of Irish junior cycle mathematics syllabuses and the content of the PISA 2003 assessment. In general, the mathematical content or concepts of the PISA items were considered to be unfamiliar to between about a third to a half of junior cycle students depending on syllabus level, and the contexts or applications and item formats to be unfamiliar to most junior cycle students. The fact that junior cycle syllabuses are a greater distance than the syllabuses of many other countries from the PISA mathematics framework and assessment items may be considered a matter for concern. Countries such as The Netherlands, where the Realistic Mathematics Education (RME) movement is particularly strong, may have had a considerable advantage in taking the PISA tests due to the proximity of its curriculum to PISA (see Oldham, 2002). Analysis of curriculum documents and materials across countries carried out as part of the TIMSS 1995 study provides further evidence of the divergence of the Irish mathematics curriculum from curricula in other countries (Schmidt, McKnight, Valverde, Houang, & Wiley, 1997). In general, more topics are covered, topics are introduced earlier, and taught for longer, at secondary grade levels in Ireland than in the typical TIMSS country, and this is so despite the limited time available for mathematics. The emphasis would appear to be on breadth of content rather than on depth of understanding.

Table 3
Percentages of PISA Items Mapping on to Junior Cycle Syllabus Content Areas

Junior Cycle Content Area (Higher and Ordinary)	Approx. % of PISA items in Content Area
1. Sets	0
2. Number Systems	10
3. Applied Arithmetic and Measure	33
4. Functions and Graphs	4
5. Algebra	5
6. Geometry	0
7. Trigonometry	0
8. Statistics	20
Not on either syllabus	30

THE IMPLEMENTED CURRICULUM: TEACHING METHODS AND METHODS IMPLICIT IN THE PISA FRAMEWORK

So far the focus has been on comparing the intended curriculum, in the form of official goals, objectives, and syllabuses of the junior cycle mathematics curriculum with the PISA mathematics framework and items. In this section, the implemented curriculum is compared with the philosophy and methodologies implicit in the PISA framework. As already noted, the rationale for the PISA mathematics framework draws on socio-linguistic theory and situated learning and authentic assessment theories, best exemplified by the Realistic Mathematics Education movement. In practice, this approach translates into classroom methodologies of the form of problem-based learning, more student-centred classes and less didacticism, the development of intra-mathematical and cross-curricular themes, and more formative assessment.[4]

Responses to a questionnaire for teachers of mathematics administered as part of the PISA survey in Ireland provide some evidence on teaching methods and emphases in the junior cycle curriculum (Cosgrove et al., 2004). The questionnaire provided data on a number of themes including: emphasis on teaching various areas of junior cycle mathematics and on particular classroom activities; teachers' views on calculator and computer use; and emphasis on PISA type mathematics. The results of the analysis of the questionnaire data indicated that (i) a small minority of total time was devoted to use of mathematical knowledge in real-life situations (4% to 5%); (ii) a minority of teachers used computers, with use largely confined to websites, spreadsheets, and presentation and word-processing software rather than mathematics-specific software; and (iii) teachers focused on the junior cycle objectives which are assessed in the Junior Certificate examination more than on objectives which are not assessed, with recall of basic facts and procedures receiving high emphasis at all syllabus levels and skills of analysis and application emphasized more at Higher level than at Ordinary or Foundation levels. These results are consistent with a view of the junior cycle mathematics curriculum as formal and abstract, with little emphasis on realistic problems or use of technology.

Efforts have also been made to obtain more direct and more detailed information about classroom practices in mathematics in different countries, for example in the TIMSS video studies (Stigler, Gallimore, & Hiebert, 2000) and the Learner's Perspective Study (Clark, 2002). The perceived benefits of video studies include getting closer to the taught curriculum in mathematics; examining mathematics teaching from fresh perspectives; revealing alternatives and provoking discussion; obtaining a detailed analysis of complex activities; and preserving classroom activity so it can be conveniently viewed and reviewed.

A videotape study of mathematics teaching in 10 post-primary schools was carried out in Ireland between 1998 and 2002 to examine the way in which teaching style and methodology impacted on the learning environment of students (Lyons et al., 2003). Two researchers observed and video-recorded two mathematics lessons in second year in each school. The mathematical content

[4] An example of a comprehensive set of mathematics curriculum materials based on the PISA philosophy has been developed through a joint project of the Freudenthal Institute in The Netherlands and the Wisconsin Center for Cognitive Studies in the USA[see *Mathematics in Context (MIC)* published by Encyclopedia Britannica at *www.mic.britannica.com*].

presented in the lessons consisted of a range of topics including applied arithmetic and measure, geometry, and algebra. Some general results of the study are as follows. First, teaching methods were mainly traditional, highly didactic, with a great deal of emphasis on teacher explanation and questions followed by student practice. The methods used were strongly focused on preparing students for examinations. Secondly, the mathematics taught was formal, mainly abstract, and divided into a series of discrete definitions and skills, generally isolated from real world contexts. There was a heavy emphasis on algebra. The focus was on procedural knowledge rather than on conceptual knowledge, and on product rather than on process. Thirdly, classroom interaction was heavily teacher initiated and dominated (96% of questions were teacher initiated). Questions were mostly lower order. The focus of questioning was on getting the right answer quickly.

These trends were consistent across schools and highlight the uniformity in mathematics teaching in Irish post-primary schools which can be attributed, in part, to the highly centralized administration of education but particularly to the dominance of the public examination system. They are also consistent with the findings of the PISA questionnaire and PISA 2003 Test Curriculum Rating Project results discussed above. It should be noted that teachers have adapted very well to teaching an extensive and abstract mathematics curriculum within the constraints of diverse student interests, public examinations, and parent and school expectations. They are given little or no opportunity, however, to experiment with other methods of approaching the teaching of mathematics, and their success is often measured by their ability to achieve good examination results.

THE ACHIEVED CURRICULUM: STUDENT ACHIEVEMENT ON THE JCE AND ON PISA

The relationship between the junior cycle curriculum and PISA from the point of view of the achieved curriculum will be considered from two perspectives: how items on the Junior Certificate examination (JCE) map on to the PISA framework, and the extent to which students' performance on the JCE matches their performance on PISA.

Mapping Junior Certificate Examination Items on to the PISA Framework

Junior Certificate examination candidates at Higher and Ordinary level sit two papers, each containing six 'long' questions, while Foundation level candidates sit one paper containing six questions; no choice is offered in any of the papers. The questions are usually divided into three parts, labelled (a), (b), (c). Part (a) typically tests recall and/or simple instrumental understanding. Part (b) generally focuses on testing procedures with which students should be familiar and which they should be able to execute fluently; it may also assess relational understanding. Part (c) is intended to address somewhat higher-order skills; it usually tests the ability to apply knowledge in fairly familiar contexts (often intra-mathematical) and/or to solve comparatively routine problems. Part (a) usually carries 20% of marks, and parts (b) and (c) 40% each. Each part of each question is usually subdivided into more specific questions or items. The PISA 2003 mathematics test contained 85 items.

Table 4
Numbers of Items in the PISA Mathematics Test and in the 2003 Junior Certificate Examinations in Mathematics

	PISA 2003 Maths Test	2003 JC Higher Level Maths Exam	2003 JC Ordinary Level Maths Exam	2003 JC Foundation Level Maths Exam
No. of items	85	77	81	32

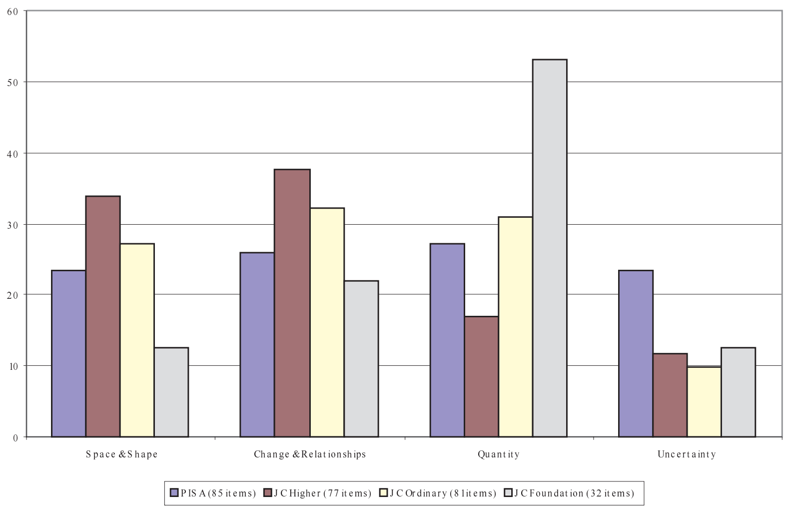
Each item in each question on each paper in the 2003 Junior Certificate examination was classified by Close and Oldham (2005) using the three dimensions of the PISA 2003 assessment framework: *overarching idea* (mathematical content), *competency cluster* (cognitive process skills), and *context* (problem situations). For comparison purposes the sub-tasks of each of the three parts (a), (b), and (c) of the six questions that students were required to answer were classified as separate items. The numbers of items emerging from this analysis are given in Table 4.

Overarching Idea. Considerable differences were found across specific examination papers in terms of the percentage of items testing each of the four overarching ideas. While in PISA there are approximately equal percentages of items across the four overarching ideas, in the Junior Certificate papers, the percentage of items assessing Quantity ranged from 16.9% (Higher level) to 53.1% (Foundation level); however, the mean of the figures is approximately 33%, which is near the PISA figure of 27 percent. A similar situation prevails for two of the other three overarching ideas, where the means across the Junior Certificate papers are not much different from those for PISA (space and shape – about 24% versus 23.5%[5];

[5] It should be noted that the percentages of items compared in the PISA space and shape category in Figure 2 include items classified as Geometry and Trigonometry in the Junior Certificate examinations. The Test Curriculum Rating Project found that the PISA space and shape items mapped mainly on to Applied Arithmetic and Measure; none mapped on to Geometry or Trigonometry (Cosgrove, Shiel et al., 2005).

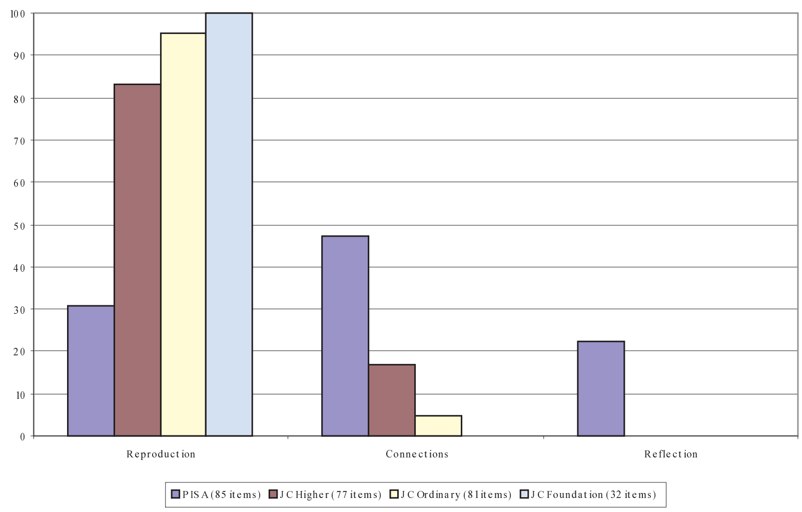
change and relationships – about 30% versus 26%). In the case of uncertainty, there are considerably more items in the PISA tests than in the Junior Certificate papers; the figures are about 11% for the Junior Certificate versus 23.5% for PISA (see Figure 2).

Figure 2
Percentage of Items Testing the Four Overarching Ideas Distributed across the PISA 2003 Test and the 2003 Junior Certificate Higher, Ordinary, and Foundation Level Examination Papers



Competency Cluster. In the mathematical process dimension, a major difference was found between the PISA tests and the Junior Certificate examination papers. Papers differed by level in the percentages of items in two of the three competency clusters (reproduction and connections). No items were classified as belonging to the reflection cluster. While in the PISA test, there is a reasonable spread of items across the three clusters, in the Junior Certificate examination papers, the percentage of items assessing reproduction ranges from 83.1% (Higher level) to 100% (Foundation level). The mean of the figures is approximately 93% across the three examination levels, compared with the PISA figure of 30.6 percent. The percentage of items assessing connections range from 16.9 (Higher level) to 0 (Foundation level); the mean of the figures is approximately 7% across Higher and Ordinary levels compared with the much larger PISA figure of 47% (Figure 3).

Figure 3
Percentage of Items Testing the Three Competency Clusters Distributed across the PISA 2003 Test and the 2003 Junior Certificate Higher, Ordinary, and Foundation Level Examination Papers



Context. There is a major difference between PISA and the Junior Certificate examination papers on the context dimension. Across the three papers, the mean percentage of intra-mathematical items is about 66%, compared to 20% for PISA, and the mean for realistic situations (including personal, educational/occupational, public and scientific situations or contexts) is about 33%, compared to 80% for PISA.

It should also be noted that the question/item formats used in the Junior Certificate examination differ substantially from those used in the PISA tests. While PISA tests use a variety of formats, including multiple-choice, short-response, and extended or constructed response with some partial credit items, the Junior Certificate examination involves mainly short-response items with students often being asked to show their work, for which partial credit could be awarded. In the extended response items on PISA, students were often required to explain their thinking which is not a feature of the certificate examinations. However, the examinations have a more detailed and comprehensive partial credit scheme which is likely to give students credit for appropriate mathematical work that would not be rewarded in the PISA marking system.

Junior Certificate Examination Performance and PISA Performance

As the final part of the comparison of the junior cycle mathematics programme with the PISA framework, the mathematics achievement of students who took the PISA tests is compared with their achievement on the Junior Certificate mathematics examination (Cosgrove, Shiel et al., 2005). For the comparison, grades (A to F) awarded in the Junior Certificate examination for Higher, Ordinary, and Foundation level were converted to a common 12-point scale [the Junior Certificate Performance Scale (JCPS) which ranges from 23 points (A on Higher level) to 1 point (F on Foundation level)].

A correlation of .75 between the PISA mathematics scores of students and their scores on the JCPS indicates a considerable overlap between the two assessments. However, this needs to be viewed in the context of the correlations among the three PISA tests of mathematics, reading, and science, which averaged around .83. Furthermore, while females outscored males on the Junior Certificate examination in mathematics, males significantly outperformed females on the PISA mathematics test. This suggests there are substantial differences in what each test measures.

The PISA achievement scale is divided into six levels, each of which is described in terms of the mathematical knowledge likely to be possessed by students at that level, with students at the highest level (Level 6) likely to be able to do the most difficult items on the test, while students at the lowest level would be unlikely to be able to do any other than the simplest items. Table 5 shows the percentages of students scoring (mean scores) at each PISA proficiency level for each of the three Junior Certificate syllabus level groups. There are, predictably, large differences in the levels of proficiency demonstrated by students taking Junior Certificate mathematics at the three levels. About one-third (33%) of Foundation level students scored below Level 1; i.e., they did not demonstrate even the most basic mathematical proficiencies associated with PISA mathematics. A further two-fifths of this group (38.5%) scored at Level 1, and 22.5% scored at Level 2. Only about 5% of students taking Foundation level scored beyond Level 2. At Ordinary level, 22% of students were at, or below, Level 1, and close to two-fifths (36%) were at Level 2. About two-fifths (42%) of Ordinary level students scored at Level 3 or higher, and less than 2% at Levels 5 or 6. At Higher level, very few students scored at or below Level 1 (1.5%), although 10.5% were at or below Level 2. About one-quarter of Higher level students (25%) scored at proficiency levels 5 and 6. These data indicate that the three levels of the Junior Certificate mathematics examination differentiate among students in terms of achievement in a manner similar to the way the PISA proficiency levels do. However, they differ in that the PISA proficiency levels describe what mathematical knowledge students at any particular level possess, which includes the knowledge and skills described at any lower level. This is not the case with the Junior Certificate examination which simply gives a grade for each student at each syllabus level, with no indication of what a student at a particular grade level or syllabus level can do in terms of mathematical knowledge and skills.

Table 5
Percentage of Students at Each Proficiency Level Cross-Tabulated with Junior Certificate Mathematics Syllabus

	<Level 1	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Higher	0.3	1.2	9.0	28.8	35.8	19.7	5.2
Ordinary	4.1	17.8	36.2	30.4	9.9	1.5	0.1
Foundation	33.4	38.5	22.5	5.5	0.0	0.0	0.0

Adapted from Cosgrove, Shiel et al., 2005, Figure 6.19.

CONCLUSION

If we accept the validity of the PISA framework, and that the performance of Irish students on PISA mathematics is not satisfactory, then it should be recognized that we may not be addressing the mathematical needs of today's students and tomorrow's citizens. In some countries (including Ireland), mathematics curricula in schools and college service courses have changed little since the 1960s when only about half an age cohort completed post-primary school. What is done in the classrooms is still largely influenced by the needs of academic mathematics and particularly third-level mathematics and mathematics service courses. This has led to a formalist and abstract mathematics curriculum which has little bearing on what most students will need by way of mathematical knowledge and skill in later years of education and of life, even if they do take mathematics-related courses. Only a small subset of the post-primary school population will need much of the mathematics they are currently learning in schools. Thus, the curriculum is not meeting either the needs of society in terms of mathematical literacy or the needs of students in terms of motivating them to learn mathematics at a level and form appropriate to their abilities, backgrounds, and learning styles. While there have been minor revisions at different times to syllabuses, examination papers, marking schemes, goals, objectives, and teaching texts, the approach has been rather haphazard, and its political nature has led to a relatively 'disconnected curriculum'. The mathematics curricula of many countries, including Ireland, need to be radically overhauled to provide for the needs of an increasingly diverse body of students. PISA, which was designed and tested by experts in mathematics education, educational statisticians, and psychometricians who work cooperatively with teachers in the field provides a framework for changing mathematics education practice as well as providing evidence for policy making.

The substantial mismatch between PISA and the Irish junior cycle mathematics curriculum can be seen in the context of a more general model of curriculum theory in which the curriculum is seen as reflecting the needs of society, learners, and the academic discipline of mathematics. The PISA mathematics framework, and curricula based on it, reflect the needs of society by focusing on mathematics as it occurs in human activity and as needed by mathematically literate citizens. It does not eschew formal academic mathematics but sees it as arising from (horizontal) mathematization of real problems and situations which can then be refined or further developed (vertical mathematization). At a more advanced level, students can be taught further design elements and procedures as the need arises in solving more complex problems as is done in third-level mathematical modelling courses. The needs of students are considered by providing them with a curriculum which motivates them by engaging them in solving problems that are meaningful and relevant.

The results of this analysis of the junior cycle mathematics curriculum from the perspective of the PISA mathematics assessment framework can be used to formulate a number of recommendations which, it is suggested, merit consideration in evaluating the appropriateness of present provision. First, there is a need to achieve greater consistency between syllabuses, examinations, teaching methods and goals, objectives, and principles. Secondly, there is a need to consider a more balanced curriculum that takes greater account of societal needs and student needs and motivation, and less account of the needs of third-level academic mathematics and mathematics service courses. Thirdly, producing new curriculum documents accompanied by limited top-down inservice is not sufficient. A more recent approach to curriculum review involves a comprehensive inservice programme starting with small groups of exemplary teachers and researchers and developing from there.[6] Fourthly, a more rigorous approach to the design and marking of examinations is required,

[6] An example of a successful programme of this nature is the LUMA project in Finland (Valijarvi, Linnakyla, Kupari, Reinikainen, & Arffman, 2002), which commenced in 1994 and is still ongoing having apparently achieved outstanding results in improving Finnish mathematics education.

broadening the scope of the examinations, and providing descriptions of what students at particular grade levels can do. Fifthly, there is a need for school and classroom-based assessment to complement the terminal examination. This would facilitate problem-based learning, group work, and continuous assessment. Sixth, the continuity from primary to post-primary school in the area of mathematics needs to be addressed. Seventh, following the approach adopted by PISA and many other countries, groups of experts in mathematics education and educational assessment and specialist teachers should be formed to review and design the mathematics curriculum in place of the present practice of ensuring representation of all stakeholders regardless of expertise. Eighth, mathematics and mathematics education should be publicized to broaden parents' attitudes to them. Finally, a pilot study should be carried out to investigate the potential benefits and difficulties of introducing a 'mathematics in context' type programme to schools.

APPENDIX

List of Basic Mathematical Topics Covered in PISA 2003 Pilot Study Items.

Quantity: Perimeter and area of 2D shapes. Volume and surface area of cube, cuboids, cylinders, cones, pyramids, spheres. Scale in maps & plans. International time zones. Buying/selling/costs/interest/discount/taxes. Ratio, proportion & percent. Convert currencies. Units of measure. Compound growth. Exponential & scientific notation. Indices. Timetables.
Shape & Space: Elementary triangle theorems/relationships. properties of 2D shapes. Geometric number patterns, Cartesian coordinates. Types of triangles. Tiling & symmetry. 2D & 3D transformations. Similar triangles. Projective geometry, Pythagoras' Theorem.
Change & Relationship: Substitution & transformation of formulae. Solution of linear equations. Connecting physical processes with graphs. Speed/time relationship. Additive & multiplicative patterns. Area/diameter relationship in circle. Area/volume relationships. Sets & Venn diagrams. Number theory - divisibility. Prime numbers. Permutations & combinations. Graphs of functions. Integers.
Uncertainty: Graphs of relationships between variables. Complex graphical data. Tables of data on two or three variables. Selecting suitable graphs for data sets. Listing outcomes of random events. Scatter plots. Frequency distributions. Mean. mode, median, quartiles.

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