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SOME ISSUES IN ELEMENTARY SCHOOL MATHEMATICS EDUCATION IN THE UNITED STATES

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There are many issues in the mathematics education of children in the United States that are being discussed and researched Of these, eight have been selected for discussion in this article formalism in contemporary programmes, individualized instruction, grouping for instruction, trends in achievement test scores, the back-to-the-basics trends, the Free School Movement, metrication, and the hand-held calculator

The year 1976 marks both the bicentennial of the independence of the United States of America and the quarter century of the so-called mathematics revolution The period beginning in 1951 and continuing into the 1970s represents the fifth major stage in the history of elementary school mathematics education in the United States It is with some of the issues which have eventuated from the many and varied forces in this period that this article is concerned But a brief look backward will give the reader some insight into the other four major periods and perhaps thereby serve to illuminate the number and magnitude of some of the problems of the present period

The first period covered the years from the wilderness beginnings through the birth of the nation in 1776 and into the first quarter of the nineteenth century (1821) In these early years the Colonial Common School curriculum consisted of reading, handwriting, religion, and an arithmetic which was limited to rote counting and writing numerals in a slate book Textbooks, as we know them, were not yet written specifically for young children The mathematical needs of adult society were essentially limited to the computational skills of the marketplace – counting, operations in whole numbers, bartering, changing shillings and pence to cents and mills, measurement of commonly used goods, etc Only a small fraction of the teen-age population attended school and college and had the opportunity to learn these and other topics within a systematically organized mathematics curriculum

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The year 1821 denotes the beginning of the second period which ended about the last decade of the nineteenth century – a total span of about three-quarters of a century. In 1821, Warren Colburn, a professor of arithmetic and mathematics at Harvard University, published his *An arithmetic on the plan of Pestalozzi*. In addition to Colburn's being strongly influenced by Pestalozzi's concern for the use of real materials and socially relevant experiences, he was also equally concerned with the need for organizing his textbook materials to reflect an inductive psychology of learning. For the first time, a significant move was made away from the deductive method of teaching. Also, for the first time during this period arithmetic was taught in the primary grades and appropriate textbooks were available. Then too, compulsory education legislation, beginning in the mid-1800s and reflecting the demand for more education for more people, brought about a substantial increase in the level of mathematical literacy by the masses.

The third period (1894-1923) is characterized by the development of scientific psychology particularly as evidenced by a new concern for a study of the developing child – the Child Study Movement. Prior to this period the psychology for school subjects was rational psychology, a branch of philosophy. From the time of Plato and Aristotle the idea that some subjects trained the mind better than other subjects was commonly held. And it was widely believed that, within arithmetic, mental arithmetic was especially helpful in strengthening the mind. Hence, mental arithmetic was a regular part of the school mathematics programme during this period.

It was the research efforts of E. L. Thorndike and R. S. Woodworth, first reported in 1901, which failed to show that one subject trained the mind better than some other subject if both subjects were equally well taught. Thus, rejecting the theory of mental discipline, it was necessary that another theory of learning be created as a basis for determining school practices. During the subsequent years, Thorndike constructed the theory of bond psychology which he identified as a new point of view concerning the general process of learning. According to this point of view, 'learning is essentially the formation of connections or bonds between situations and responses and that the satisfyingness of the result (later called the law of effect) is the chief force that forms them. (16, p. v.)'

One result of Thorndikean stimulus-response (S-R) psychology was that each mathematical topic was broken into small cognitive bits or atoms, hence S-R bond psychology was also called atomistic psychology (as well as associationistic, connectionistic and mechanistic) This period was primarily one of modest advances or at least substantial changes in the psychology of methodology But the question of *what* mathematics should be taught was still answered by appealing to the need for teaching socially significant mathematics rather than the structure of the discipline itself

The fourth period, 1923-1951, was essentially the period of the Progressive Education Association (PEA) movement Officially organized in 1919, and guided by John Dewey, the goals of the PEA were carried out through three rather distinct thrusts the child-centered thrust of the 1920s, the social-reform thrust of the 1930s, and the scientific psychology thrust of the 1940s The Association officially disbanded in the post-World War II period claiming that it had accomplished what it set out to do

Throughout this period, the theory of the curriculum was in an ebb-tide, flood-tide situation with the two counter forces being the expressed needs of the child and the 'scientifically' determined adult needs of society But in actual practice only a few schools could be found that reflected the former theory of curriculum

From the point of view of a psychology of teaching and learning, the most significant development of this period was the conceptualization and effective research and expository writing of the meaning theory of arithmetic by William A Brownell According to this theory, it was not sufficient for the child to learn only the arithmetic that he had immediate need for or only the arithmetic he might need in later adult social situations Rather, the primary goal should be the systematic and sequential teaching that would result in the learning of the structure, the organization, and the inner relationships of the subject itself – a harbinger of things to come in the form of the 'new mathematics '

The fifth and present period of development can be thought of as beginning in 1951 or 1952 From this point in time until the Russians sent Sputnik into space in 1957 there was a very marked increase in the number and volume of the critics of the school mathematics programme No longer were they willing to see the content determined by the incidental, often accidental, needs of the child, or limited to the meagre uses of mathematics by the mathematically semi-literate average adult Increasingly, the meaning theory gained in acceptance and school use

Funded by federal agencies and private foundations, several individuals or groups prepared new mathematics programmes, first for the secondary school level, then in the late 1950s for the elementary school level Although differing widely in actual content, organization and completness, all these programmes were alike in that they were based on a theory of curriculum which emphasized the logical structure of the discipline – the formalism of the subject

All programmes claimed to be organized around powerful, abstract organizing structures – integrating ideas – such as the concept of sets and the properties of the real number system In completeness, these programmes, developed during the 1960s and early 1970s, varied from those of the prestigious School Mathematics Study Group (SMSG) which was the product of a large, well-balanced interdisciplinary team to the seemingly capricious assembling of a few disparate and largely inappropriate topics which reflected the idiosyncratic interests of the individual writer of the programme or project The former programmes had a significant educational impact on subsequently published school programmes, the latter only the trival and transitory effects of small often cult-like groups

In the latter part of this period and up to the present it became increasingly clear that good mathematics by itself was not enough to make a good mathematics programme *How* one taught was at least as important as *what* one taught The theoretical work and research studies of a group of cognitive psychologists, most notably Jean Piaget, Jerome Bruner and Robert Gagné, as well as the continuing influence of William Brownell, have made substantive contributions to how elementary school mathematics programmes are now taught These developments do not mean that many problems do not remain to confront mathematics educators Eight contemporary issues facing mathematics teachers will now be briefly discussed

CONTEMPORARY PROGRAMMES

All too often the programmes developed in the era of the 'new math' were characterized by a heavy emphasis on formalism, formalism in language and formalism in the development of each topic For instance, considerable use was made of the properties of the real number system to rationalize the algorithms By way of an illustration, in the 'old math' a standard technique for processing the number sentence $3 - 3/4 = n$

consisted of 'inverting the divisor and multiplying' Hence $3 - 3/4 = 3 \times 4/3 = 12/3 = 4$ No rationalization of the rule was taught

In the 'new math' this same number sentence would be processed in any one of several ways A common development was

$$3 \div 3/4 = \frac{3}{\frac{3}{4}} = \frac{3}{\frac{3}{4}} \times 1 = \frac{3}{\frac{3}{4}} \times \frac{\frac{4}{3}}{\frac{4}{3}} =$$

$$= \frac{3 \times \frac{4}{3}}{\frac{3}{4} \times \frac{4}{3}} = \frac{\frac{12}{3}}{1} = \frac{4}{1} = 4$$

Herein the child could, if well taught, readily comprehend the use of the identity element for multiplication (one) as well as the use of the renaming of 1 as $\frac{4}{3} - \frac{4}{3}$ and the use of 1 in the form $\frac{3}{4} \times \frac{4}{3}$

Excessive use of formalism in the definitions of terms, in operations on numbers, and in the processing of numerals (algorithms) of the 'new math' concerned many thoughtful mathematics educators Some believed the more rigorous of these programmes to be appropriate for only the top third or top fourth of all the children and youth who were using them Van Engen (17), mathematician and mathematics educator, expressed the thoughts of many when he said 'most certainly there is reason to question the degree of formalism that is creeping into the elementary school Furthermore, the rapid pace of the more usual programme is questionable'

Nor was the concern limited to the mathematics educators Cognitive psychologists witnessing the harm being done not only through ill-fitting mathematics textbooks, but also ill-fitting reading and language arts programmes spoke out Benjamin Bloom (2) expressed his concern thus, 'there seems to be little reason to make learning so difficult that only a small proportion of the students can perservere to mastery' And David Ausubel (1) stated that 'much of children's alienation from school is a reflection of the cumulative effects of a curriculum that is too demanding'

Recognizing the strong reactions from the teachers trying to use the textbooks with average and below-average ability children, some publishers, beginning in the late 1960s, either modified the rigor in subsequent editions or produced new textbooks that were less formalistic and more socially relevant.

INDIVIDUALIZED INSTRUCTION

Several publishers as well as teachers in several local school systems attempted to solve the problem of fitting the programme to the varying abilities of the children by developing what are euphemistically called individualized programmes They are more correctly labelled *self-paced* programmes All children do the same work, only the pace varies

The best advertised and perhaps most costly to produce is *Individually Prescribed Instruction – Mathematics (IPI-Math)* (11) This programme was originally developed by Research for Better Schools Inc at the University of Pittsburgh and is now published commercially *IPI-Math* has been researched from both a theoretical and an empirical point of view

A discussion of the theoretical rationale of *IPI-Math* which applies equally well to similar self-paced programmes appears in *Overview and Analysis of School Mathematics, Grades K-12* (14) It was written by the National Advisory Committee on Mathematical Education and is referred to as the *NACOME Report* It was authorized and financed by the prestigious Conference Board of the Mathematical Sciences and the National Science Foundation The *NACOME Report* (14) focuses on four major weaknesses of individualized or self-paced programmes Firstly, such programmes do not (cannot) match instruction to learning styles The only individualized characteristic is pace, and the only objective seriously sought is computational skill Problem solving skills, conceptual foundations and attitudes, which should be part of any good programme, are missing Secondly, by emphasizing, even requiring, that the children work alone, the programme militates against pupil-pupil interaction and teacher-pupil interaction Yet interaction is essential to integration and the development of positive self-concepts And 'considering the limited time devoted to mathematics in the instructional programme, it is clear that in a narrowly defined "individualized programme" the amount of time a teacher spends with each student is very small (14, p 59) '

The third major criticism by *NACOME* of *IPI-Math* and similar self-paced programmes is that 'the emphasis on testing until mastery tends to lead students to shallow learning of "local" rules and to emphasize only low-level skills There is a tendency to learn just those rules which allow passing the post-test (14, p 59)' Fourthly, such programmes require a great quantity of record-keeping It has been shown that as much as 83% of the teacher's time is devoted to managing the system, leaving less than 20% for developmental lessons, for diagnosis and for reteaching

From the theoretical point of view, Joseph Lipson (12, p 60), developer of *IPI-Math*, recently stated that 'the programme did *not* produce the dramatic gains that had been hoped for because the programme, and many like it, was built on false assumptions'

Schoen (15) has summarized the findings of nearly one hundred studies of self-paced mathematics programmes Six of his eighteen summary findings seem particularly relevant Firstly, overall mathematics achievement is likely to be less in an individualized programme than in a traditional one In fact, achievement rate appears to decrease each year of individualization Secondly, individualized programmes are more expensive than traditional ones Thirdly, excessive amounts of test taking, isolation of children and lack of a mechanism for students to unify the ideas to be learned are some problems mentioned by researchers Fourthly, the educational quality of the pupil-teacher interaction in the individualized classrooms is very poor, consisting mostly of procedural matters Fifthly, the present techniques for diagnosis and prescription are ineffective And sixthly, student self-scoring is very unreliable, the pre-set performance criteria for the units may not be valid, and multimedia instructional options are rarely used even when they are available

Concern for the educational quality of *IPI-Math* and similar self-paced programmes has reached to the very highest levels of professional organizations Most significantly, at the 1975 Annual Meeting of the National Council of Teachers of Mathematics, the Delegate Assembly recommended for consideration by the executive board the following resolution

> That the National Council of Teachers of Mathematics should recognize the magnitude of the problems arising from the widespread and often precipitous adoption of programmes described as 'individualized instruction' in school mathematics by undertaking, without delay, a short-term study of such programmes, with results to be published as soon as possible

GROUPING FOR INSTRUCTION

Although the overly rapid spread of self-paced programmes is causing great concern in the United States, by far the most common practice for teaching mathematics is through whole class instruction This is characterized by a single developmental lesson for all followed by a practice or application time during which all children do as much of a single assignment as possible Obviously a single lesson and a single practice assignment cannot fit well all of say thirty children of widely varying abilities any more than a single size shoe can fit well all of those same children.

To avoid the pitfalls of both self-paced instruction at one extreme and whole class grouping at the other, and while recognizing that each of these has some limited – even necessary – uses, the sensible solution would seem to be grouping children according to their ability to learn mathematics

In recent years, over thirty plans for grouping children for instructional purposes have been developed Each was put forward as 'the' solution to complex problems of individual differences Some have all but disappeared, some are in a neonate stage Hundreds of research studies have been carried out to evaluate the effectiveness of these plans Most of these studies are trivial and fail to meet the canons of scientific research Outstanding among such studies is the superb study by Goldberg, Passow and Justman (10) They investigated the effectiveness of fifteen plans for narrowing the ability range on cognitive and affective behaviour Three thousand children in grades 5 and 6 (ages 10-11 and 11-12) in eighty-six intact classes in predominantly middle-class schools in New York City were used Five ability levels were designated from below-average to gifted The testing programme included several subject matter areas and several affective (non-academic) variables

Goldberg *et al* stated their general conclusion in predominantly middle-class elementary schools, narrowing the ability range in the classroom will, by itself, in the absence of carefully planned adaptations of content and method, produce little positive change in the academic achievement of pupils at any ability level Concerning the affective variables, the study found no evidence that such ability grouping is associated with negative effects on self-concept, attitudes toward school or other affective variables. They conclude that 'ability grouping, *by itself*, has no important effect on the academic achievement of students (10, p v)' 'It is . *what* we teach that matters, not how we sort out the students It is on the differentiation and appropriate selection of content and method of teaching that the emphasis must be placed (10, p 169) '

SOME TRENDS IN ACHIEVEMENT TEST SCORES

In recent years there has been substantial concern evidenced over the sharp decline in scores on various standardized mathematics achievement tests Publishers of the most widely used standardized achievement tests for elementary and junior high schools (ages 6 to 14) report declines in test scores during the norming procedures On the highly regarded Scholastic Aptitude Tests (SATs) which are administered to high school students and used as criteria for selection and admission to college, the mean scores on both the quantitative section and the verbal section have shown very marked declines over the period 1962 to 1975 On the quantitative section, the mean score in 1962-63 was 502 (on a range of 200 to 800) In 1974-75, the mean score was 472 And the percentage of students scoring above 600 declined from 20 2% to 16 4% On the verbal section, the drop was from 478 to 434 In scores above 600, the decline was from 14 6% to 8 9%

There also has been concern over the level of performance shown by children, youth and young adults on the first administration of the mathematics section of the National Assessment of Educational Progress (NAEP) programme (7) The *NAEP* is a large scale measurement and evaluation programme of the abilities of 9-, 13-, 17-year olds, and young adults ages 26 to 35 The complete programme covers several subject matter areas The areas are examined on a rotating basis with each area being tested every four to five years The first administration of the mathematics tests was in 1972-73 The subtests covered six content areas numbers and numeration, measurement, geometry, variables and relationships, probability and statistics, and consumer mathematics

While comparative data will not be available until the next administration, descriptive data for the first administration are available Some of the test items have been released for public use Examples of performance on a number of items are as follows

1 Only 40% of the 9-year olds did this addition correctly

$	3 09
	10 00
	9 14
	5 10

2 Only 33% of the 13-year olds could find the correct answer to the problem If John drives at an average speed of 50 miles per hour, how many hours will it take to drive 275 miles?

3 Only 1% of the 17-year olds and 16% of the young adults could balance a cheque book

4 Problem A housewife will pay the lowest price per ounce for rice if she buys it at the store which offers

	13	17	adult
12 ounces for 40 cents	13%	10%	4%
14 ounces for 45 cents	9%	8%	5%
1 pound, 12 ounces for 85 cents	25%	34%	39%
2 pounds for 99 cents	46%	46%	47%
I don't know	6%	3%	4%

The results speak for themselves, indicating the present-day pathological state of mathematics learning and instruction.

Virginia H Knauer, Director of the US Office of Consumer Affairs reacted to the findings of the report saying 'This report brings home the hard fact that consumers do not have the math skills necessary to solve the day-to-day problems we face in today's economy Yet the importance of developing these is indisputable.'

THE 'BACK-TO-THE-BASICS' TREND

The data sampled above, supported by the large numbers of more casually done studies on the state and local educational levels, have caused a substantial ground-swell in the form of a 'back-to-the-basics' trend Parents' groups, writers of letters to newspapers, sensationalist writers in the slick magazines and supplements to Sunday newspapers, these and others have taken up the cause in an unorganized but apparently significant fashion. This effort is similar to that during and immediately following the World War II years. During that period, members of the US military expressed strong concern for the lack of basic mathematical competencies on the part of young draftees and the citizenry in general

Following the war, well-organized groups, such as the Council for Basic Education, carried on sustained criticisms of the effectiveness of the schools

Their cause was eventually lost in the mounting anxiety created by Sputnik and in the subsequent enthusiasm for a 'new math', and 'new science', a 'new reading', a 'new social studies', etc These new programmes varied widely in quality and in appropriateness for the differing abilities of the learners They attained their apogee in the mid-1960s The results of the standardized tests for children and youth, the Scholastic Aptitude Tests for college-bound youth, and the NAEP results (as mentioned above) have brought a new wave of concern and care for the firmness of the foundations (9) This return, for the fourth time in the century to a philosophy of curriculum which stresses essentialism or utilitarianism, has caused Joseph Featherstone to comment

> In one sense, it seems discouraging that our efforts to improve practice have not gone beyond the formulations of the Progressives The general lack of cumulative development makes a good deal of our educational reform seem terribly faddish In what I sometimes think of as the United States of Amnesia, we keep rehearsing the dilemmas of the past, and I suppose we will continue to start from scratch each generation until we develop a sixth sense of the past to add to our other five senses (6, p 3)

This trend will also run its course In the meantime, it is aided and abetted by any number of local, state and national committees, some self-appointed, others formally appointed, to study the causes and cures for the apparent decline in performance on school-type mathematics tests

THE FREE SCHOOL MOVEMENT

The free school movement in today's elementary and secondary schools has its historical origins, usually unknown to today's proponents, in the Progressive Education Movement of the 1920s and its demise in the early 1950s It was best conceptualized and explicated by John Dewey in his book, *The school and society*, published in 1899 The Progressive Movement was not the creation of a few crackpots or eccentric minds Rather it was a serious and scholarly attempt to bring the schools into a laminar flow, as it were, with a society undergoing rapid transformations by the forces of democracy, science, industrialism and the insights derived from the social psychological sciences of human development In a word, the school should be congruent with the progressive changes in social and political thought The school should mirror society As noted above, the Movement had three thrusts a child-centered thrust, a social-reform thrust, and a scientific thrust

The 'free school movement of today is usually called 'open education' or 'informal education' The leading philosopher-historian of the Progressive Educational Movement is Dr Lawrence A Cremin, President of Teachers College, Columbia University (5) He dates the renaissance of the Movement in the form of open education with the publication of A S Neill's *Summerhill* in 1960

Cremin recently compared the two movements and found the free school movement wanting

> What is most striking, perhaps, in any comparison of the two movements is the notoriously atheoretical, ahistorical character of the free school movement in our time The present movement has been far less profound in the questions it has raised about the nature and character of education and in the debates it has pursued around those questions The movement has produced no John Dewey, no Boyd Bode, no journal even approaching the quality of the old *Social Frontier* And it has been far less willing to look to history for ideas Those who have founded free schools have not read their Francis W Parker, or their Caroline Pratt or their Helen Parkhurst, with the result that boundless energy has been spent in countless classrooms reinventing the pedagogical wheel (4, p 72)

Quite often the open school is incorrectly and naively conceptualized as the architectural modification of a school building Instead of, say, four separate classrooms each with twenty-five children, the walls of the four classrooms are removed to make one large classroom with one hundred children Openness is confused with open space Even more trivial and incorrect, open education is viewed as a number of novel and unrelated cute learning experiences such as tie-dying of cloth, macrame work and cooking foods native to other countries While each of these may enliven the classroom, they have little to do with the fundamental nature and purposes of progressive education

What has the rebirth of progressive education to do with mathematics education in the USA? Some critics are quick to assign the cause of the decline in achievement test scores, noted above, to open or informal education When mathematics is taught with decreased concern for systematic teaching, sequential teaching, and mathematically-structured teaching, it is reasonable to assume that there will be a decrease in learning, retention of learning, transfer of learning and verbal problem solving ability However, the number of schools that can be truly called open are few Hence, it is quite unlikely that open education, by itself, is a primary cause of the decline

Also, open education has not been the cause of any measurable improvement in the quality of our educational programmes Again, in the words of Cremin

> the movement has had immense difficulty in going from protest to reform, to the kinds of detailed alternative strategies that will give us better educational programmes than we now have (4)

Open education has succeeded in some few schools in providing children with a more humane learning environment – good in itself – while in some other schools the learning environment has gone from some reasonable control to one of wild excitement short of anarchy In the near future, perhaps some well-designed studies using, say, the *NAEP* data will provide evidence over time of the effect of openness on mathematical achievement

METRICATION

Teachers and publishers are trying to 'tool up' for making the metric system, *Systeme International d'Unites* (S I), a natural part of the instructional programme Primary responsibility will fall on the mathematics teachers, but gradual integration of metrication in science and in other subject matter areas can be anticipated

The slogan for this new trend in America is *Think Metric* and appears everywhere from automobile bumper stickers to lapel buttons to professional journals The educational significance of the slogan is, of course, that teachers should require children and youth to think and work *directly* with units of metric measure rather than think with English units and convert to metric units using some formula such as that for converting Celsius to Fahreheit

How much metrication should children learn? The National Council of Teachers of Mathematics has recently published a list of competencies to be acquired (presumably, by children of average ability) at the end of grade six (age twelve) Other lists of competencies have been established for ages nine and fifteen (13) At the end of the sixth grade, students should be able to

(i) Select a unit model for each of the units, metre, decimetre, centimetre, litre, and kilogram, and measure lengths to the nearest whole number of millimetres

(ii) Use a metrestick (or other rule) with millimetre markings to measure line segments and linear objectives to the nearest tenth of a centimetre, tenth of a decimetre, and tenth of a metre, use the kilometre to describe experience-related travel distances, and apply the following equivalencies

10 mm = 1 cm
100 cm = 1 m
10 cm = 1 dm
1000 mm = 1 m
10 dm = 1 m
1000 m = 1 km

(iii) Make a direct reading of the weight (mass) measure of an object to the nearest tenth of a kilogram from a scale, read the measure of a liquid or granular substance in a graduated container to the nearest ten millilitres

(iv) State applications for each metric unit with which they have basic familiarity, from areas such as commerce, industry, science, and the arts

(v) Identify the unit name associated with each of the symbols mm, cm, dm, m, km, g, kg, ml, l, and °C and, in most cases, reverse the process

(vi) State that the linear dimensions of the standard model of a square centimetre and a cubic centimetre are 1 cm by 1 cm and 1 cm by 1 cm by 1 cm, respectively, make a similar statement for the dimensions of the standard models of a square metre and a cubic metre

(vii) Recognize and apply the following relationships

1 metre is a little more than a yard,
1 kilometre is a little more than ½ mile,
1 kilogram is a little more than 2 pounds,
2 5 cm is about 1 inch, and 1 litre is a little more than 1 quart

This is the extent of conversions between the two systems recommended for the interim changeover period

(viii) Relate 0 C and 100 C to the freezing and boiling temperatures of water, identify 37 C as 'normal' body temperature, and identify temperatures in the human 'comfort zone' (about 22 C to 25 C)

(ix) Estimate distances up to 5 metres in whole metres and lengths up to 10 centimetres in whole centimetres

(x) Estimate volumes up to 5 litres in whole litres and estimate 250 millilitres (approximately 1 cup)

(xi) Compute sums and differences of measures expressed in decimal form such as

1 36 m	4 200 1
+2 49 m	–1 600 1
3 85 m	2 600 1

(xii) Express personal heights in centimetres and personal weights (mass) in kilograms

Obviously it will take many years of competent child study and programme development before we can presume to make a reasonably good cognitive 'fit' between any one of the topics above and the ability of the child to learn that topic In time, however, we should be able to say that topic X can be readily learned by above average ability in grade three, and that some topic can be readily learned by slow learners in say grade six Also, we can look forward to very substantial changes in the extensive programme in operations on fractional numbers now taught in grades five and six

THE HAND-HELD CALCULATOR

Within the past few years the cost of the least expensive hand-held calculator (mini-calculator) has decreased from about $100 to as little as $10 The ready availability of this device and its perceived use in eliminating the need for learning the conventional algorithms has caused school personnel and parents to call for guidelines in school use and in school-related home use

The National Council of Teachers of Mathematics after considerable study of the issue released the following position statement

> The mini-calculator should be used in imaginative ways to reinforce learning and to motivate the learner as he becomes more proficient in mathematics.

Glenadine Gibb (8, p 1), President of the National Council of Teachers of Mathematics, made a more helpful statement

> One guideline for all of us, however, is that we do not use the calculator until our students have developed a concept of number, a system for naming numbers, and an understanding of the meaning and processes of the basic operations – that is, until our students understand what the calculator is doing for them

The *NACOME Report* (14) identified five changes which will take place if mathematics education takes 'full advantage' of the mini-calculators

> First, the elementary school curriculum will be restructured to include much earlier introduction and greater emphasis on decimal fractions, with corresponding delay and de-emphasis of common fraction notation and algorithms
>
> Second, while students will quickly discover decimals as they experiment with calculators, they will also encounter concepts and operations involving negative integers, exponents, square roots, scientific notation and large numbers – all commonly topics of junior high school instruction These ideas will then be unavoidable topics of elementary school instruction
>
> Third, arithmetic proficiency has commonly been assumed as an unavoidable prerequisite to conceptual study and application of mathematical ideas This practice has condemned many low achieving students to a succession of general mathematics courses that begin with and seldom progress beyond drill in arithmetic skills Providing these students with calculators has the potential to open a rich new supply of important mathematical ideas for these students – including probability, statistics, functions, graphs, and coordinate geometry – at the same time breaking down self-defeating negative attitudes acquired through years of arithmetic failure
>
> Fourth, for all students, availability of a calculator does not remove the necessity of analyzing problem situations to determine appropriate calculations and to interpret correctly the numerical results
>
> Fifth, present standards of mathematical achievement will most certainly be invalidated in 'calculator classes' An exploratory study in the

Berkeley, California public schools indicated that performance of low achieving junior high students on the Comprehensive Tests of Basic Skills improved by 1 6 grade levels simply by permitting use of calculators (14, pp 41-2)

As with the impending impact of metrication, it will require much research combined with conventional wisdom before reasonably good educational decisions and school practices can be developed to facilitate the integration of the hand-held calculator into classrooms containing children of widely varying abilities

CONCLUSION

In this brief paper I have tried to present an overview of a few of the more pressing issues in elementary school mathematics education in the USA While it is necessary to isolate such issues for purposes of research and discussion it should be kept in mind that the classroom is a holistic integrated learning environment Many of these issues interact with each other For instance, how can the degree of formalism of a mathematics programme be tailored to both the individual's ability to learn and to how the children shall be grouped for instruction? How much of the metrication programme should be presented to slow children? To bright children? And with what degree of formalism? Should all children learn to perform the operations on integers with a unit of metric measurement using the hand-held calculator? Or should the programme be limited to the non-negative rationals in decimal form for children of average ability? Should slower learning children who cannot understand the algorithm for dividing decimals be allowed to learn without understanding by using the little black box – the hand-held calculator?

In the same way, *mutatis mutandis*, we could ask many questions which point up the integrated nature of the eight seemingly disparate issues discussed above That these, and other issues, will be with us for some time is clear But not all have the same life expectancy We may hope that the weight of present research and conventional wisdom will quicken the end of self-paced instruction as the method for all children On the other hand, it may be a long time before it can be said that all teachers group children for instructional purposes in elementary school mathematics.

We have not discussed so very many questions on which research is now available but in which school practices lag years behind This writer and Dr Leroy G Callahan of the State University of New York at Buffalo very recently published the fourth edition of *Elementary school mathematics A guide to current research* (3) This book contains a summary of research on very many educationally significant questions m mathematics education which are not discussed in this paper

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