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ASSESSMENT OF PROJECT MATHS AT JUNIOR CERTIFICATE LEVEL: AN EXPLORATORY STUDY USING THE PISA AND TIMSS ASSESSMENT FRAMEWORKS

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Project Maths, a new mathematics curriculum, was implemented in schools in Ireland between 2008 and 2015. This paper describes an analysis of the content, cognitive processes and contexts underpinning Junior Certificate mathematics examination questions set for students in Third Year in 2003 and 2015, using the frameworks of the PISA and TIMSS international studies. Despite a significantly increased reading load for students, albeit with support in the form of scaffolding at all examination levels, the Junior Certificate mathematics examination continues to over-emphasise lower-order processes, at the expense of higher-level thinking, as defined by TIMSS and PISA, while there has been a small increase in the proportion of items presented in practical, though not necessarily realistic, contexts. It is concluded that the 2015 Junior Certificate mathematics examination more closely resembles TIMSS than PISA. The findings are discussed in the context of a lack of evidence on the effects of Project Maths on student performance beyond an initial evaluation.

The revised mathematics curriculum for post-primary schools (Project Maths), which seeks to emphasise deep conceptual understanding and problem solving in real-life contexts, has been the focus of considerable debate in Ireland as educators and students have dealt with changes to syllabi, new teaching methods, and changes to assessment. Some commentators have claimed that the OECD Programme for International Student Assessment (PISA) has had a disproportionate impact on Project Maths (e.g., Kirwan, 2015). This paper examines how the content, processes and contexts underpinning the Junior Certificate mathematics examination have changed since the implementation of Project Maths, with reference to the assessment frameworks for the Trends in International Mathematics and Science Study (TIMSS) and PISA. Changes in the reading load required of students are also examined. These analyses are preceded by an overview of Project Maths.

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OVERVIEW OF PROJECT MATHS

The development and implementation of the Project Maths curriculum between 2008 (when it was introduced into 24 initial or pilot schools) and 2015 (when all strands were assessed in both the Junior and Leaving Certificate mathematics examinations)[1] represented the first 'root and branch' revision of the post-primary mathematics curriculum in Ireland since the advent of the 'New' or 'Modern' Mathematics curriculum, implemented between 1964 and 1973. The 'New Maths', a worldwide movement at the time, emphasised unifying themes such as Sets, Relations, and Functions to promote better understanding of the conceptual and logical structure of mathematics. In Ireland, dissatisfaction with the 'New Maths' curriculum developed in the 1970s, and led to a number of revisions. These mainly comprised deletions and additions of specific topics in Number and Algebra, simplification of Geometry, clarifications of rationales and objectives, the addition of a third syllabus level (along with the Higher and Ordinary levels) for lower-achieving students and the introduction of calculators into teaching and learning (Oldham 2001; DES/NCCA 2000, 2002).

Prior to Project Maths, a number of studies identified potential difficulties with mathematics education in post-primary schools in Ireland. These included:

- a didactic pedagogy with mathematics being taught in a procedural fashion with relatively little emphasis on problem solving (Lyons, Lynch, Close, Sheeran & Boland, 2003);
- an over-reliance on rote learning and a lack of deeper understanding of basic mathematics concepts (SEC, 2003a, 2005);
- average performance on international assessments of mathematics (e.g., Beaton et al., 1996; OECD, 2001, 2004);
- low levels of basic mathematics knowledge shown by some students proceeding into higher education (O'Donoghue, 2002); and
- negative attitudes towards mathematics on the part of adults and children in Irish society in general (NCCA, 2012).

A focus over the past 20 years on building Ireland as a knowledge economy and a recognition of the importance of science, technology, engineering and mathematics (STEM) have also been factors. Indeed, the importance of mathematics in the context of STEM has been recognised in the report of a government-appointed STEM Education Review Group

[1] All strands were assessed at Leaving Certificate level in 2014.

(2016), in which strategies for the further development of the teaching and learning of STEM subjects in schools have been laid out.

The Focus of Project Maths

On its website, the National Council for Curriculum and Assessment (NCCA) describes Project Maths as 'an exciting, dynamic development in Irish education. It involves empowering students to develop essential problem-solving skills for higher education and the workplace by engaging teenagers with mathematics set in interesting and real-world contexts' (NCCA, 2016). Elsewhere, the NCCA (2008, p. 1) has described Project Maths as placing 'greater emphasis on developing students' essential numeracy skills and on the use of contexts and modern applications of mathematics that are relevant to students' present and future needs'. In the same report, it describes Project Maths as a 'root-and-branch' revision of the mathematics curriculum at both Junior and Leaving Certificate levels.

According to the current Junior Certificate syllabus (DES/NCCA, 2013, p. 6), the aims of Project Maths are to:

- develop the mathematical knowledge, skills and understanding needed for continuing education, for life and for work;
- develop the skills of dealing with mathematical concepts in context and applications, as well as in solving problems;
- support the development of literacy and numeracy skills; and
- foster a positive attitude to mathematics in the learner.

These broad aims are accompanied by objectives designed to support the development of mathematical proficiency – conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. The key mathematics content areas are identified as Statistics and Probability, Geometry and Trigonometry, Number, Algebra, and Functions.

The primary structural differences between the revised Project Maths curriculum at Junior Certificate level and its predecessor are the absence of a specific course for Foundation level, and the introduction of learning outcomes for each topic within each strand. In terms of content, the most marked difference is the introduction of Probability for both Higher and Ordinary levels and the expansion of Statistics to cover topics such as Sampling (Higher level only) and Statistical Reasoning (e.g., awareness of misuses of statistics). In addition, the content for each strand in the revised curriculum concludes with a section relating to synthesis and problem-solving skills. This includes learning outcomes such as the justification of conclusions and communication of mathematics, both verbally and in written form. Notably, an additional feature of the Project Maths syllabus is the inclusion of a supplement dealing with Plane Geometry that is separate from the rest of the curriculum, with an approach and general philosophy that is more formal and reminiscent of the previous curriculum.

As well as changes to curriculum, Project Maths envisaged changes to instruction. According to the NCCA (2012, p. 10),

> Project Maths, informed by international trends, develops key skills by promoting a 'collaborative' culture where mathematics is seen as a network of ideas which teacher and students construct together. Learning is seen as a social activity in which students are challenged and arrive at understanding through discussion. Teaching is seen as a non-linear dialogue in which meanings and connections are explored, misunderstandings are recognised, made explicit and students learn from them.

The NCCA (2012, p. 18) also emphasised that students are 'encouraged to think about their strategies, to explore possible approaches and evaluate these, and so build up a body of knowledge and skills that they can apply in both familiar and unfamiliar situations'.

The implementation of Project Maths included a significant attempt to upgrade the skills of mathematics teachers to embrace new, discovery-based teaching methods designed to enhance conceptual understanding and problem solving (see, for example, the resources for teachers available at *http://www.projectmaths.ie/*).

Response to Project Maths

Project Maths has had a mixed reception. The Irish Mathematics Teachers Association (IMTA, 2012) noted that insufficient detail was provided on aspects of course content, giving rise to uncertainty as to whether certain topics were included or not. IMTA also referred to an over-representation of Statistics and of construction in Geometry, and, at Leaving Certificate level, the absence of vectors. IMTA expressed concern over the literacy demands of Project Maths, arguing that unnecessarily difficult or elaborate language was used in curriculum materials and examinations. Drawing on a survey of teachers in pilot and non-pilot schools conducted as part of PISA 2012 in Ireland, Cosgrove, Perkins, Shiel, Fish and McGuinness (2012) reported more frequent use of Information and Communication Technologies (ICTs) in mathematics classes in pilot schools, and more positive changes in learning and assessment, though teachers in pilot schools were less confident in their teaching. Teachers in pilot schools also argued that the problems presented to students in classroom and assessment contexts contained more text and greater linguistic complexity than was the case prior to Project Maths, when teaching and learning mathematics were more formal and less contextualised.

A report by academics at University College Cork (Grannell, Barry, Cronin, Holland & Hurley, 2011) questioned whether Project Maths could adequately prepare students for the depth and breadth of third-level mathematics courses. The report argued that reform in mathematics could have been achieved by training teachers on the existing curriculum, rather than, as the report's authors saw it, lowering the standard of material in the curriculum. Grannell et al. and Kirkland (2012) strongly questioned the absence of certain topics (e.g., vectors, matrices, sequences and series, aspects of calculus) that had been dropped in the transition to Project Maths, though mainly at Leaving Certificate level. Kirkland also argued that, at the same level, there is too much emphasis on applications and not enough on mathematical foundations including calculus and linear algebra.

Lubienski (2011) noted substantive differences between textbooks claiming to be based on Project Maths, with one of the two she examined following a didactic pattern (presenting boxed formulae and examples for students to follow) and the other structuring a sequence of investigations through which students discover the formulae. In a study that compared a broader range of Project Maths textbooks at Junior Certificate level, O'Keefe and O'Donoghue (2011, p. v) noted that the textbooks 'displayed a genuine attempt to match the intentions of Project Maths, but no one textbook met all the needs of Project Maths'. They also found a mismatch between curriculum expectations (such as integration of content strands) and textbook expectations, minimal emphasis on the integration of ICTs into teaching and learning activities, and lack of consistency between textbooks in the extent to which teaching for understanding and problem solving was promoted.

Systemic Changes Coinciding with Implementation of Project Maths

An initiative designed to improve take-up of mathematics at Higher level in the Leaving Certificate examination has been to provide bonus points to students achieving a grade D or higher in Leaving Certificate Higher level mathematics from 2012. This has resulted in increases in the proportions of students taking Higher level mathematics in both the Leaving and Junior Certificate, with uptake at Leaving Certificate increasing from 22% in 2012 to 27% in 2015, and uptake at Junior Certificate increasing from 48% to 55% over the same period (SEC, 2012, 2015a)[2]. According to the *National Strategy to Improve Literacy and Numeracy 2011-2020* (DES, 2011), the target for participation at Junior Certificate Higher level mathematics is 60% by 2020.

Another initiative has been to introduce a Professional Diploma in Mathematics that enables 'out-of-field' teachers to meet the Teaching Council of Ireland's requirements to teach mathematics in post-primary schools.

Evaluation of Project Maths

A number of sources can be drawn on to evaluate the effects of Project Maths on student performance in and attitudes towards mathematics at Junior Certificate level. These include an official evaluation of Project Maths commissioned by the NCCA, the performance of students in Ireland on PISA before and during implementation of Project Maths, and performance on the Junior Certificate mathematics examination. Additional information is available in the report of the Chief Examiner for Junior Certificate mathematics (SEC, 2015b).

The official evaluation of Project Maths was conducted by a group from the National Foundation for Educational Research in England (Jeffes et al., 2012, 2013), using tests and attitudinal scales based on released items from international studies. Rather discouragingly, it reported that 'overall, schools following a greater number of strands, or schools having a greater experience of teaching the revised syllabuses, does not appear to be associated with any improvement in students' achievement or confidence' (2013, p. 5). Moreover, there was a lack of evidence of the processes underlying Project Maths in students' written classwork, with an apparent focus on content rather than process. The evaluators did, however, note that their evaluation occurred at a relatively early point in the implementation of Project Maths.

All pilot Project Maths schools were included in the sample of schools in Ireland that participated in PISA 2012[3]. Students in these schools achieved

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[2] In 2011 (the year before bonus points were introduced), 46% took the Junior Certificate mathematics examination at Higher level, while 16% took the Leaving Certificate mathematics examination at Higher level (SEC, 2011)

[3] By 2012, the number of pilot Project Maths schools had fallen to 23, as one school had amalgamated with a non-Project Maths school, and was no longer considered to be a pilot school. The contribution of Project Maths pilot schools to Ireland's overall performance on PISA 2012 was weighted down, to reflect the representation of students in Project Maths schools in the overall population of students.

higher mean scores than students in non-pilot schools (most of whom had not studied under Project Maths) on each PISA mathematics content area and process, and on the overall performance scale, but differences were not statistically significant (Merriman, Shiel, Cosgrove & Perkins, 2014). Worryingly, students in pilot schools had significantly higher levels of anxiety about mathematics (as measured by PISA) than their counterparts in non-pilot schools, perhaps because they were the first to experience changes to the Junior Certificate examination arising from the transition to Project Maths.

The Junior Certificate mathematics examination can also be used as a basis for comparing performance over time (i.e., before and after full implementation of Project Maths), though it is recognised that questions and marking schemes change from year to year. Table 1 shows the percentages of Junior Certificate students achieving grades A to NG (no grade) in 2003, 2012 and 2015 at each syllabus level.

Table 1
Percentages of Junior Certificate Students Achieving Each Grade in Junior Certificate Mathematics, by Level (2003, 2012, 2015)

Year	A	B	C	D	E	F	NG
Higher Level							
2003	17.2	33.6	28.6	17.0	3.1	0.5	<0.1
2012	15.1	31.4	32.7	18.0	2.5	0.3	<0.1
2015	11.3	31.2	32.2	21.1	3.7	0.5	<0.1
Ordinary Level							
2003	9.2	31	31.3	20.8	5.8	1.8	<0.1
2012	14.3	33.9	28	17.0	5.0	1.5	0.2
2015	7.4	28.3	34.5	23.8	4.9	0.9	0.1
Foundation Level							
2003	15.4	37.8	29.5	13.6	3.2	0.4	<0.1
2012	17.1	34.2	30.1	15.6	2.3	0.7	<0.1
2015	14.6	35.9	30.2	16.2	2.7	0.4	<0.1

Source: SEC (2003b, 2012, 2015a).

Table 1 shows that, from 2003 to 2015, there was a decrease in the percentage of students achieving A grades in Higher level mathematics (from 17% to 11%). There was a marginal decrease in the percentage of B grades, accompanied by slight increases in C and D grades. These trends should be interpreted in light of the changing proportions of Junior Certificate students opting for the Higher level examination. In 2003, 41% of students took the Higher level examination. This increased to 48% in 2012 and to 55% in 2015. It is likely that the Higher level cohort in these years included students who, in previous years, would have taken the Ordinary level examination. Even though these are likely to be the higher-achieving students from that group, it could be expected that overall performance on the Higher level examination would decline somewhat. A similar trend might be expected at Ordinary level, again assuming that stronger Ordinary level students began opting for the Higher level papers. Table 1 shows a slight decrease in Ordinary level A and B grades between 2003 and 2015, with a spike in A grades in 2012 (the first year in which bonus points were on offer at Leaving Certificate level, but before the first cohort of students in non-pilot Project Maths schools was examined on Project Maths at Junior Certificate level).

Table 1 shows reasonable stability at Foundation level across the selected years with only minor fluctuations. This is perhaps surprising, given that the percentage of students opting for the Foundation level paper dropped from 12% in 2003 to 6% in 2015. One might expect this change to result in weaker students remaining at Foundation level. However, the lack of a differentiated Foundation level course at Junior Certificate means that these students now cover the same content as Ordinary level students. This might counter any potential negative trends in the results of the Foundation level cohort.

Few studies have looked at actual implementation of Project Maths in classrooms. Lubienski (2011) noted a tension between an explicit emphasis on problem solving in Project Maths, which teacher-respondents in pilot schools in her survey linked to 'realistic mathematics education' (even though Lubienski observed that many of the problems they used were not set in the real world), and teacher practice, which favoured demonstration over discovery and investigation.

International Studies of Mathematics and Project Maths

Since the implementation of New Maths, increased globalisation and technological change have led to the expansion and development of international studies, including the TIMSS and PISA studies.

Ireland participated in the first of the TIMSS four-yearly assessments of students in Second Year (Grade 8 internationally) in 1995 (Beaton et al., 1996), and, after a break of 20 years, in 2015 (Fourth class or Fourth grade students in Ireland participated in the 2011 TIMSS study). TIMSS places an equal emphasis on mathematics and science in each four-year cycle.

Ireland has participated in all six cycles of PISA's three-yearly assessments of 15-year-olds since 2000. In PISA 2003 (Cosgrove, Shiel, Sofroniou, Zastrutzki & Short, 2005) and PISA 2012 (Perkins, Shiel, Merriman, Cosgrove & Moran, 2013), mathematics was the major domain. In the other years, it was a minor domain, with reading literacy or science designated as the major domain.

TIMSS focuses on assessing performance on a common core of mathematics content, most of which is covered in the mathematics curricula of the participating countries. PISA mathematics, on the other hand, focuses on assessing mathematics with tasks which are embedded in practical, preferably 'realistic', contexts which can vary in their familiarity to students across the countries involved. Although half of the items in the TIMSS tests are also embedded in practical contexts, such contexts are mainly minimal and contrived compared with the PISA items which are embedded in more substantial and more realistic contexts. This reflects differing philosophies and goals, with TIMSS seeking to assess the systematically-taught and well-practised mathematical knowledge and skills students have acquired from schooling (Mullis, Martin, Ruddock, O'Sullivan & Preuschoff, 2009; Mullis & Martin, 2013) and PISA seeking to assess how well students can use the knowledge and skills they have acquired to solve problems arising in life – also referred to as *mathematical literacy* (OECD, 2013).

In the TIMSS 1995 mathematics assessment, students in Second Year in Ireland achieved a mean score of 527 on the international scale which has a centre point of 500 (standard deviation = 100). Of the 45 countries participating, only seven scored significantly higher than Ireland (Beaton et al. 1996). These results seemed to be viewed as satisfactory and did not provoke any concern nationally although the scores on the content areas of Geometry and Measurement were relatively low (51% and 53% respectively) compared with Number and Data/Probability (65% and 69%).

In the first three cycles of PISA, Ireland performed at a level that was not significantly different from the international mean of 500 (set in 2000, and again in 2003). In 2009, Ireland's mean score dropped to 487 points (i.e., it was significantly below the OECD average)[4], but improved again to 503 in

[4] Performance on reading literacy in Ireland also declined in PISA 2009. Among the factors associated with the declines in performance were disengagement from the tests by students, changes to the design of the tests, and the method used to scale achievement and link

2012, when it was significantly above the OECD average for the first time. However, Ireland's improved standing relative to the OECD average can be attributed to a drop in the OECD average itself, rather than to an improvement among students in Ireland, compared to their performance between 2000 and 2006. It is notable that, when performance on mathematics content areas and processes was reported on by the OECD in 2012, students in Ireland achieved mean scores that were significantly above the corresponding OECD average scores on Change and Relationships, Quantity and Uncertainty and Data, but significantly below the OECD average on Space and Shape, which includes aspects of Geometry, Visualisation of 2- and 3-D Shapes, and Applied Measurement. Female students in Ireland, in particular, struggled with Shape and Space items.

In TIMSS 2015, students in Second Year in Ireland achieved a mean score (523) that was significantly higher than the international average, and a ranking of 9th among 39 participating countries (Clerkin, Perkins & Cunningham, 2016). Six countries, five from Southeast Asia (Singapore, Korea, Chinese Taipei, Hong Kong China and Japan) and the Russian Federation, performed at a significantly higher level than Ireland, while the mean scores of students in Kazakhstan, Canada, the United States, England and Hungary were not significantly different from Ireland's. In PISA 2015, 15-year-olds in Ireland achieved a mean score of 504 and a ranking of 18th among 70 participating countries and economies (Shiel, Kelleher, McKeown & Denner, 2016). While the mean score of students in Ireland was significantly above the average for OECD countries, 16 countries, including Estonia, the Netherlands, Denmark, Finland and Slovenia, achieved significantly higher mean scores than Ireland. Students in PISA 2012 and 2015 in Ireland had similar mean scores in mathematics. It could be that the effects of Project Maths on performance in PISA 2015 are masked by other issues – in particular, the transition from paper-based to computer-based assessment. The fact that PISA mathematics was a minor assessment domain in 2015 also means that changes on specific content areas and processes, which could have arisen as a consequence of Project Maths, were not reported on.

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performance across cycles, though it has not been possible to quantify the precise contributions of these factors to Ireland's lower mean scores (Cosgrove, 2015).

Some commentators (e.g., Kirwan, 2015; Kirwan & Hall 2016) have argued that PISA in particular has had an undue influence on Project Maths. According to Kirwan (2015), Project Maths closely follows the PISA conceptual framework. However, Kirwan also acknowledges that Project Maths is comprised of two distinct approaches – the abstract, symbolic mathematics of sections of the pre-existing curriculum, as well as a PISA-like approach to pedagogy and real-life problem solving. Indeed, in an analysis of the 2015 Junior Certificate Examination, she reported that just under one-half of questions at each level (Higher, Ordinary and Foundation) could be categorised as 'abstract/symbolic' and just over one-half as 'real-life' problems.

TIMSS, PISA and Curriculum Matching

Each TIMSS study has involved a *curriculum matching* exercise in which subject experts in each country analyse how well the TIMSS assessment framework and tests match that country's mathematics curriculum for the age level or grade involved. In the TIMSS 1995 curriculum matching exercise, Ireland's mean percentage score on the TIMSS mathematics items that teachers identified as being on the curriculum for Second Year students in Ireland was 59% versus 58% on all the items on the TIMSS 1995 test (Beaton et al., 1996). This reflected the closeness of the mathematical content of the TIMSS test and the Irish mathematics curriculum, before the introduction of Project Maths. The TIMSS 1995 study also showed that the Irish Junior Certificate syllabus covered more content at this level than many other participating countries.

In Ireland, but not internationally, a somewhat different matching exercise was carried out as part of PISA 2003 when mathematics was the main assessment domain. Given that all PISA items are problems set in 'realistic' contexts, teacher-raters were asked to rate the familiarity of the items to students in terms of mathematical concept, application context, and response format. The majority of the mathematical concepts involved in the PISA test items were rated as being familiar to Irish 15-year-olds, especially students who had taken the Higher and Ordinary level examinations. However, it also found that 30% of the concepts in PISA items were not covered in the Junior Cycle syllabi and that a number of topics on the Junior Cycle mathematics syllabi were not assessed in the PISA test – Sets, Indices, Functions, Synthetic Geometry, Coordinate Geometry, and Trigonometry. Also, the majority of item contexts (whether personal, occupational, public or scientific) and response formats (multi-choice, short or extended response) were rated as being not familiar to students taking Higher, Ordinary, and Foundation levels (Cosgrove et al., 2005).

A further analysis compared the 2003 Junior Certificate mathematics examinations with the PISA 2003 mathematics framework dimensions (Close & Oldham, 2005; Oldham, 2006; and Close, 2006).

Regarding the four overarching ideas of PISA[5] (that is, Quantity, Change and Relationships, Space and Shape, and Uncertainty), the analysis found considerable differences in the percentages of items (sub-questions) testing each overarching idea for each of the three Junior Certificate examination levels. However, when averaged across the three examination levels, the percentage of items was not much different from the percentage of PISA items assigned to each overarching idea (c.25 %) with the exception of Uncertainty where the average percentage of Junior Certificate items was 11%, compared with 23.5% for the PISA test. This was mainly due to there being no items on Probability and few on Statistics in the 2003 Junior Certificate examination papers, a situation that changed with the advent of Project Maths.

Major differences were found between the 2003 Junior Certificate examination items and 2003 PISA items on the three PISA competency clusters (Reproduction, Connections and Reflection), with the vast majority of Junior Certificate items at all three examination levels classified as Reproduction (Higher – 83%; Ordinary – 95% and Foundation – 100%), a very small percentage classified as Connections (17% – Higher; 5% – Ordinary; 0% – Foundation), and no items classified as Reflection. The corresponding PISA figures were Reproduction – 31%, Connections – 47% and Reflection – 22%. These data were consistent with the Irish results on the PISA mathematics proficiency scale. This scale has six levels ranging from Level 1 and 2 (students at these lower levels can answer the easiest and most routine of problems, mainly Reproduction), through intermediate Levels 3 and 4 (with a mix of all three competencies) to Levels 5 and 6 (students at these upper levels can answer the most difficult and complex problems on the test, mainly Connections and Reflection). About a quarter of Higher level students and 5% of Ordinary level students achieved at PISA Levels 5 and 6, while about two-thirds of Ordinary level students achieved Levels 2 and 3. Just 5% of Foundation level students achieved beyond Levels 1 and 2.

[5] PISA's 'overarching ideas' (content areas) and competency clusters (processes) are described in detail in the methodology section of this paper.

The findings for the context dimension (where items are classified as mainly personal, occupational, public or scientific) also showed substantial differences between the Junior Certificate examinations and the PISA test, with more than half of the items on the Higher, Ordinary, and Foundation level examinations being intra-mathematical (that is, context-free), and the remaining items embedded in a practical context, compared with the PISA test in which all the items were embedded in a practical context. The Junior Certificate examination item contexts were generally minimal and more contrived compared with the more substantial and 'realistic' contexts of the PISA problems.

Readability of Mathematics Tests and Examinations

As noted earlier, a number of surveys on the implementation of Project Maths raised issues about the volume of reading in mathematics textbooks and in examinations, and the linguistic complexity of the problems. This reflected a transition from mainly context-free or abstract mathematics in the past, to a situation in which many questions were embedded in context under the new syllabus (though this was already happening to some extent in the Junior Certificate Ordinary and Foundation level papers prior to Project Maths). The concerns of teachers may relate to a perception that some students can adequately apply their mathematics in situations in which questions are relatively context free, but are at a disadvantage when asked to read a short text, abstract the underlying problem in mathematical terms, solve it, and, in some instances, interpret the results in terms of the original problem. Eivers (2010) expressed this somewhat differently. She attributed the heavy reading load in PISA mathematics (and science) items to a need by PISA to contextualise items and present them in real-life contexts, and argued that this contributed to 'construct-irrelevant variance' that made it difficult to assess whether low performance arose from reading or mathematical difficulties.

A number of studies have been implemented to measure the readability of mathematics assessments, including TIMSS and PISA. Mullis, Martin and Foy (2013) identified four factors that might render mathematics items difficult: number of words (with more words indicative of a greater reading load); vocabulary complexity (with specialised vocabulary likely to be more difficult); complexity of symbolic language such as numerals, symbols and abbreviations; and density or complexity of visual displays such as geometric shapes and figures, models and diagrams. They categorised TIMSS 2011 Grade 4 items into those deemed to have a 'high reading demand', 'medium reading demand', and 'low reading demand'. Fifty percent of items in the Number content area were deemed to have low reading demand, while 85% in the Data content area were judged to have 'high reading demand'. Whereas 59% of items categorised as 'Reasoning' (the highest order process skill) were judged to have high reading demand, 61% categorised as 'Knowing' (the lowest order process skill) were deemed to have a low reading demand. Data for Ireland indicated that, in general, performance in mathematics was related to students' reading proficiency according to a standardised measure of reading (the test underpinning the Progress in International Reading Literacy Study) which was administered in conjunction with TIMSS 2011, and involved the same schools and students. Hence, on average, lower-performing readers in Ireland performed least well on items categorised as having high or medium reading demands. High achievers in reading, on the other hand, performed at about the same level across items with high, medium and low reading demands. It is unclear if these same patterns are evident in TIMSS Grade 8.

Researchers in mathematics education have applied traditional readability formulae to mathematics tests, while acknowledging their drawbacks. Such formulate are typically designed for use with relatively long, continuous texts rather than short items accompanied by diagrams. In addition, differences in estimates of reading difficulty may arise from the application of different formulae. King and Burge (2015) sought to address these issues by applying multiple formulae – mostly developed in the US – to clusters of items administered in PISA 2012. Their results, when averaged across formulae, showed that mean US grade levels ranged from 7.5 to 10.9. These correspond to reading ages extending from 12.3 to 15.5 years. This suggests that large proportions of students taking the PISA 2012 tests (those with reading ages below 15 years) might expect to find the reading aspect of the mathematics items to be especially difficult. Using the Flesch-Kincaid formula, Shiel, Cosgrove, Sofroniou and Kelly (2001) reported an average reading difficulty of US Grade 6.8 for mathematics items administered in PISA 2000, with a range of 3.7 grade levels. This is marginally lower than the average of the values reported by King and Burge using the same formula for item clusters in PISA 2012 mathematics (Grade 8.6, range 2.1 grade levels). This may suggest that PISA mathematics items have become more complex, in terms of readability, over time. To our knowledge, there are no detailed data available on the readability of the Junior Certificate mathematics examination.

AIMS OF THE STUDY

The aims of the current study are: to classify Junior Certificate mathematics items in the examination papers for 2003 and 2015 (i.e., before and after the full implementation of Project Maths) through the lens of the TIMSS and PISA mathematics assessment frameworks; and to evaluate the readability of Junior Certificate mathematics items in the examination papers for 2003 and 2015 and compare these with readability levels for the TIMSS and PISA assessments.

The first aim is designed to investigate whether the examination has changed in accordance with the aims of Project Maths. The second is designed to ascertain if the reading levels underpinning the Junior Certificate mathematics examination have changed under Project Maths, and how these compare with current international studies.

METHODOLOGY

In line with these aims, aspects of the methodology relating to the analysis of the content, processes and contexts underpinning the Junior Certificate mathematics examination papers in 2003 and 2015 are described, along with the approach taken to evaluating the readability of the same examination papers, and of the TIMSS and PISA mathematics items.

Examination Paper Analysis – Content, Process and Context

The present study focused on the Junior Certificate examination papers in 2003 and 2015 as examples of papers before and after the introduction and implementation of Project Maths. The 2003 papers were chosen because a similar classification exercise was previously carried out on these papers by Close and Oldham (2005)[6]. In addition, as in 2012, the 2003 cycle of PISA assessed mathematics as the 'major assessment domain'. The 2015 papers were chosen as a comparison as these were the most recent papers with available results after Project Maths had been fully implemented.

Junior Certificate mathematics is currently examined at three levels: Higher level, Ordinary level and Foundation level. There are two separate papers each

[6] As noted earlier, Close and Oldham (2005) conducted an analysis of the 2003 Junior Certificate papers using the PISA 2003 framework. The PISA framework analysis for the 2003 papers was re-done for the current study to ensure consistency in the application of the classification systems across both assessment years (2003 and 2015).

for Higher and Ordinary levels, and one for Foundation level. Consequently, the study included 10 papers in total (five for each year). The aim was to classify the questions in each examination paper in terms of the main components of the TIMSS 2015 framework (Grønmo, Lindquist, Arora, & Mullis, 2013) and the PISA 2003 and 2012 frameworks (OECD, 2003, 2013) [7]. The relevant characteristics of these frameworks are summarised below.

TIMSS Mathematics Framework

TIMSS mathematics has four Content Domains, each with a number of sub-topics:

- *Number* – whole numbers, fractions, decimals and integers, ratio and proportion;
- *Algebra* – expressions and operations, equations and inequalities;
- *Geometry* – geometric shapes, geometric measurement; location and movement; and
- *Data and Chance* – characteristics of data sets, data interpretation, and chance.

The framework also outlines three Cognitive Domains:

- *Knowing* – recalling and understanding mathematical facts and concepts, and performing computations and straightforward algebraic procedures;
- *Applying* – routine and familiar problem-solving, and representing and modelling problem situations;
- *Reasoning* – analysing and generalising from relationships, drawing conclusions, making justifications, and problem-solving in new or unfamiliar contexts.

PISA Mathematics Framework

The PISA mathematics framework has three dimensions: Context, Content and Competence. All items are presented in an applied context which is either personal, occupational, societal or scientific.

There are four Content Categories:

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[7] For the purposes of this study, the PISA 2012 framework was used to classify items by Content (it defines the Content Areas in the same way as the 2003 framework). The 2003 Competency Clusters were used, rather than the Competency Clusters introduced in 2012. The older Competency Clusters were deemed to be more suited to the style of items in the Junior Certificate examinations and are more consistent with the TIMSS Cognitive Domains. The 2015 framework for mathematics (OECD, 2016) is similar to the 2012 framework.

- *Change and Relationships* – understanding types of change and recognising when they occur in order to use suitable mathematical models to describe and predict change;
- *Space and Shape* – understanding perspective, creating and reading maps, transforming shapes with and without technology, interpreting views of three-dimensional scenes from various perspectives, and constructing representations of shapes. This category draws on Geometry, Spatial Visualisation, Applied Measurement and Algebra;
- *Quantity* – understanding measurements, counts, magnitudes, units, indicators, relative size, and numerical trends and patterns, and employing number sense, multiple representations of numbers, mental calculation, estimation, and assessment of reasonableness of results;
- *Uncertainty and Data* – knowledge of variation in processes, uncertainty and error in measurement and chance.

A range of content topics is embedded in these categories including: functions; algebraic expressions; equations and inequalities; co-ordinate systems; relationships within and among geometrical objects in two and three dimensions; measurement; numbers and units; arithmetic operations; percentages; ratios and proportions; counting principles; estimation; data collection; data variability and its description; samples and sampling; and chance and probability (OECD, 2013).

PISA also classifies items by cognitive process or 'Competency Cluster'. Similar to TIMSS, these competencies have a hierarchical structure:

- *Reproduction* – knowing facts and common problem representations, performing procedures, applying standard algorithms and manipulating expressions;
- *Connections* – integrating, connecting, and problem solving in situations that are more than routine but placed in somewhat familiar contexts;
- *Reflectio*n – planning and implementing solutions to more multi-faceted and original problems and some reflection on this process.

It is worth noting that while there are some elements common to the cognitive processes in the two frameworks, there are some important differences. In particular, the Reproduction cluster in PISA can be described as the 'reproduction of practised knowledge', which covers the category of Knowing in TIMSS but also some of Applying, such as routine problem-solving. Furthermore, the Reflection Cluster in PISA involves 'advanced' reasoning, abstraction and generalisation in novel contexts, which often requires a higher cognitive demand than some Reasoning items in TIMSS.

Classification Process

Within each Junior Certificate paper, each part of a question (a i, a ii, etc.) was treated as a separate item[8]. Each item was classified by the first two authors of this paper according to the dimensions outlined above. In addition, each item was assigned a Strand and Topic from the Junior Certificate Syllabus. Classifications were carried out independently and all disagreements were recorded and discussed until an agreement was reached. Very few disagreements arose in relation to the content or context of the items. By comparison, more disagreements occurred where judgements were made about the processes involved i.e., in identifying TIMSS Cognitive Domains and PISA Competency Clusters. This is not surprising as determining the primary process required to answer a test item is, by its nature, a more subjective exercise. Initial agreement rates per examination ranged from 74% (TIMSS classifications for Foundation level in 2015) to 97% (PISA classifications for Foundation level in 2003). Most examinations had an agreement level above 80% (see Appendix, Table A1).

During the classification process, a number of issues arose due to differences in mathematical content between the Junior Certificate examination and the international assessments. For example, neither TIMSS nor PISA usually include items involving factorisation of algebraic expressions, or solving quadratic equations. However, given that these topics are merely extensions of material that is covered in both assessments, these items were classified as Algebra and Change and Relationships, respectively. A different example related to Set Theory, which featured in most of the Junior Certificate papers that were classified, but does not appear in either the TIMSS or PISA frameworks. The TIMSS framework clearly sets out the content areas that are included in the assessment. Therefore, items on sets were classified as 'Not covered' under the TIMSS Content Domain category. In contrast, the PISA framework is not curriculum-based and is less clear about the specific mathematical content that might appear in the assessment. Consequently, the lack of an explicit reference to Set Theory does not preclude related content from appearing in PISA. Given this, items on Sets

[8] While each question part was treated separately from the others, in many cases these 'items' were not independent i.e., some question parts relied on information acquired from solving an earlier part.

were classified as belonging to Change and Relationships in PISA. In cases where items were considered 'Not covered', all such items were nevertheless classified by cognitive process, as the same classification principles could be applied. In the case of some types of items, the first author contacted the TIMSS and PIRLS International Study Centre for their advice on classifications (see Appendix, Table A2).

One additional issue arose during classification in relation to PISA contexts. The authors felt that the PISA categories were not entirely fit for purpose for this study, as the practical contexts used in the Junior Certificate examinations are not as rich as those used in PISA, therefore rendering a direct comparison somewhat misleading. To address this, contexts were additionally classified as falling into one of three categories: Practical (Authentic), Practical (Minimal) and Intra-mathematical. Items were classified as 'Authentic' if situated in a practical context that was carried through the item i.e., where students had to engage with real-life concepts in order to provide a solution. For example, items on tax credits and net pay were classified in this category. Items in the 'Minimal' category were characterised by pseudo-contexts that did not affect the solution and were effectively the same as an intra-mathematical item. For example, one item required students to find the surface area of a 'ball in the shape of a sphere'. This context was not considered to be authentic as it did not require students to engage with a real-life concept in any meaningful way.

Readability Analysis of Examination Papers and TIMSS Tests

An analysis of the readability of Junior Certificate mathematics examination papers administered in 2003 and 2015, and the TIMSS 2015 mathematics test for Grade 8 was also conducted. This involved generating data on word count (i.e., a measure of reading load), number of sentences, average number of words per sentence, and average number of complex words (words with three or more syllables). Seven readability formulae (Flesch reading ease, Flesch-Kincaid Grade Level, Fog Scale, Smog Index, Coleman-Liau Index, Automated Readability Index, and Linsear Write Formula) were applied to text files based on each examination paper, yielding scores (US grade level equivalents) for each formula. An overall measure of readability for each Junior Certificate examination paper was obtained by taking the mean of the seven readability measures.

Prior to applying counts and formulae, all title information, general instructions and item numbers were removed, diagrams were deleted (though text within diagrams was retained), and functions and formulae were deleted and replaced with the string 'function' or 'formula', as readability formulae are not designed to assess the complexity of these elements. Three booklets, selected at random from the 14 booklets administered in TIMSS 2015 at Grade 8, were analysed in a similar way. Unlike the Junior Certificate examination papers, where between 2 hours (Ordinary and Foundation levels) and 2.5 hours (Higher level) are allocated to students, 45 minutes are allocated to each of mathematics and science in TIMSS. The recent readability analysis of PISA 2012 mathematics by King and Burge (2015) meant that it was not necessary to analyse PISA items in the same depth.

RESULTS

This section reports on the results of the analysis of the Higher, Ordinary and Foundation level Junior Certificate examination papers for 2003 (pre-Project Maths) and 2015 (Project Maths fully implemented). The results are organised in line with the dimensions of the TIMSS and PISA frameworks i.e., by Content Domain (Overarching Idea in PISA), Cognitive Domain (Competency Cluster in PISA), and Context (not a formal dimension of TIMSS framework) and by Junior Certificate year and examination level. Following this, the outcomes of the readability analyses are presented.

Content Domains

Table 2 presents percentages of items by content domain for the Junior Certificate papers for 2003 and 2015 compared to the equivalent percentages for the mathematical content dimensions of the TIMSS and PISA frameworks. The most noticeable change in content, between 2003 and 2015 across the three examination levels, is the increase in the percentage of items relating to Data and Chance in TIMSS and Uncertainty in PISA. For example, the proportion classified as Data and Chance doubled for both Ordinary level (from 10% to 20%) and Foundation level (from 13% to 26%). This change corresponds to the greater emphasis on Statistics in Project Maths and the introduction of Probability into the Junior Certificate curriculum for the first time. This brings the proportion of items on Data and Chance/Uncertainty more into line with those of TIMSS and PISA. Noticeable also is the substantial increase in the percentage of Algebra (Change and Relationships in PISA)[9] items in the

[9] As noted earlier, Change and Relationships in PISA can also include Sets.

Higher level examination between 2003 and 2015, taking them out of line with TIMSS (29% to 37%) and PISA (32% to 45%) proportions.

Table 2

Comparison of Item Percentages for the TIMSS and PISA Tests, and the 2003 and 2015 Junior Certificate Higher, Ordinary and Foundation Level Examinations, by Content Domain

TIMSS Content Domain	TIMSS % Items	JC 2003 % Items	JC 2015 % Items	PISA Content Domain	PISA % Items	JC 2003 % Items	JC 2015 % Items
			Higher Level				
Number	30	12	5	Quantity	25	21	8
Algebra	30	29	37	Change & Rels.	25	32	45
Geometry	20	37	26	Space & Shape	25	29	23
Data & Chance	20	12	19	Uncertainty	25	12	19
Not covered	--	10	13	Not covered	--	7	5
			Ordinary Level				
Number	30	20	22	Quantity	25	23	26
Algebra	30	23	21	Change & Rels.	25	34	28
Geometry	20	29	27	Space & Shape	25	26	25
Data & Chance	20	10	20	Uncertainty	25	10	19
Not covered	---	18	10	Not covered	---	7	2
			Foundation Level				
Number	30	47	26	Quantity	25	53	26
Algebra	30	16	19	Change & Rels.	25	22	23
Geometry	20	19	26	Space & Shape	25	13	26
Data & Chance	20	13	26	Uncertainty	25	13	26
Not covered	---	6	5	Not covered	---	0	0

Note: Aggregate percentages may not sum to 100% due to rounding.

This increase at Higher level was matched by a corresponding decrease in Geometry items according to TIMSS (37% in 2003 to 26% in 2015) and in Space and Shape items according to PISA (29% to 23%). Also worth noting is the decrease in items classified as Number (TIMSS) at both Higher and Foundation levels. This percentage was already small in the 2003 Higher level paper (12%) and fell to 5% in 2015. At Foundation level, the proportion almost halved, from 47% to 26 percent. Small percentages of items in the

2003 and 2015 examination papers (about 11% across all examination papers) were not covered by the TIMSS framework, and to a lesser extent the PISA framework, including items on Sets, Trigonometry, and some theorems in Geometry. This could suggest a broader scope to the Irish Junior Certificate mathematics curriculum but it should be noted that the Junior Certificate examinations are administered to Irish students at the end of their Third Year in secondary school. In contrast, TIMSS is administered to students near the end of their Second Year and PISA to 15-year-olds who are distributed across a range of grade levels. Thus the Junior Certificate examination allows for more time to cover the additional topics compared with TIMSS. There are also some differences in the way content areas are defined in the Project Maths syllabus as compared with the international frameworks. For example, some elements of the 'Number – Applied Measure' topic in Project Maths would be considered Space and Shape items in PISA and Geometric Measurement items in TIMSS.

Cognitive Domains

In the earlier study by Close and Oldham (2005), the biggest difference between the PISA test and the 2003 Junior Certificate examination papers was in the competency clusters. This finding is reaffirmed in the present study. Table 3 shows that, for both the 2003 and 2015 Junior Certificate examinations at all three levels, the majority of items, from 70% at 2015 Higher level to 100% at 2003 Foundation level, belong to the Reproduction Cluster. There are few items, from 0% at 2003 Foundation level to 27% for 2015 Higher level, in the Connections Domain, and generally no items in the Reflection Domain, apart from 2% in the 2015 Higher level examination. On the other hand, if the percentages of the 2003 and 2015 examination items are compared with reference to the TIMSS cognitive domains, we can see that these are broadly similar for the Knowing domain at Higher level (32% and 35%), while the percentage of Knowing items at Junior Certificate Ordinary level increased from 29% to 47%, and the proportion at Foundation level remained more or less the same (59% and 60% respectively). The percentages of Applying items dropped at all three levels between 2003 and 2015, with the largest drop occurring at Ordinary level (from 65% to 43%). The percentages at both Higher (49%) and Ordinary (43%) levels are now broadly in line with TIMSS (40%). Surprisingly, in light of Project Maths' emphasis on developing adaptive reasoning, the percentages of Reasoning items at all three examination levels lagged well behind the TIMSS figure of25%, though some changes occurred since 2003 (for example, 12% of Foundation level items were classified as Reasoning in 2015, compared with 0% in 2003).

Table 3
Comparison of Item Percentages for the TIMSS and PISA Tests, and the 2003 and 2015 Junior Certificate Higher, Ordinary and Foundation Level Examinations, by Cognitive Domain

TIMSS Cognitive Domain	TIMSS % Items	JC 2003 % Items	JC 2015 % Items	PISA Cognitive Domain	PISA % Items	JC 2003 % Items	JC 2015 % Items
			Higher Level				
Knowing	35	32	35	Reproduction	25	79	70
Applying	40	56	49	Connections	50	21	27
Reasoning	25	12	17	Reflection	25	0	2
			Ordinary Level				
Knowing	35	29	47	Reproduction	25	90	86
Applying	40	65	43	Connections	50	10	14
Reasoning	25	6	10	Reflection	25	0	0
			Foundation Level				
Knowing	35	59	60	Reproduction	25	100	84
Applying	40	41	28	Connections	50	0	16
Reasoning	25	0	12	Reflection	25	0	0

Note: Aggregate percentages may not sum to 100% due to rounding.

Context

Part of the intention of Project Maths reform was to place more emphasis on using mathematics to solve problems set in practical and preferably realistic contexts. Context is a dimension of the PISA framework and test and the 2012 framework classifies the dimension into four categories – personal, occupational, societal, and scientific. The contexts of all the PISA items are considered to be authentic and realistic rather than contrived or pseudo-realistic. The TIMSS framework has no context dimension but about half of the items include some sort of practical context, so the remaining items can be considered to have an intra-mathematical context. Table 4 shows the results of classifying the 2003 and 2015 items for each examination level as having an intra-mathematical context, an authentic practical context or a minimal context (as defined in the methodology section above).

Table 4
Comparison of Junior Certificate Mathematics Examination Percentages by Context Category and by Examination Level

Context Category	JCH 2003 % Items	JCH 2015 % Items	JCO 2003 % Items	JCO 2015 % Items	JCF 2003 % Items	JCF 2015 % Items
Intra-mathematical	64	52	56	48	59	44
Practical - Authentic	29	48	41	46	41	49
Practical - Minimal	7	0	2	6	0	7

JCH = Junior Certificate Higher level; JCO = Junior Certificate Ordinary level; JCF = Junior Certificate Foundation level.

The data show that more or less half of the Junior Certificate items at the three examination levels in both 2003 and 2015 were set in practical (mainly authentic) contexts, with a slight decrease in intra-mathematical contexts from 2003 to 2015. This is similar to the percentage of TIMSS items that have practical contexts. The exception is the 2003 Higher level examination where only 36% of items were classified as practical.

Readability

Table 5 shows that, at Higher level, the number of words students were expected to read in Paper 1 increased between 2003 and 2015 from 765 to 1335 (a 75% increase). At Ordinary level, the word count in Paper 1 increased from 662 to 1240 words (an 87% increase), and at Foundation level (common paper), the increase was from 530 to 1027 words (a 94% increase). There was an increase from 835 to 1620 words on Higher level Paper 2 (94%), while the number of words on Ordinary level Paper 2 increased from 935 to 1051 (12%). The average difficulty of the texts that students were expected to read (based on an average of seven readability formulae) remained about the same at Higher level (US Grade 4) and Ordinary level (US Grade 2). There was an increase at Foundation level (from US Grade 0/beginning reader level to Grade 2). It is remarkable that, for the most part, the examiners have held the average difficulty of the text constant, even though the word count has increased substantially. The proportion of difficult words, defined as words with three or more syllables, was also about the same at each level in 2003 and 2015 ranging from 4% (in 2003 Foundation level) to 9% (in 2003 Higher level Paper 2 and in 2015 Higher level Paper 1).

Table 5
Readability Measures for Junior Certificate Mathematics Examination Papers in 2003 and 2015

Exam Paper	No. of Item Parts	No. of words	No. of sentences	Avg. no. words per sentence	No. of complex words (%)	Flesch-Kincaid grade level	Avg. read-ability grade
2003 HL Paper 1	32	765	58	13	61 (8)	5.0	4
2015 HL Paper 1	45	1335	135	10	123 (9)	4.1	4
2003 HL Paper 2	41	835	78	11	73 (9)	4.3	4
2015 HL Paper 2	39	1620	160	10	131 (8)	4.4	4
2003 OL Paper 1	40	662	76	8	35 (5)	2.6	2
2015 OL Paper 1	41	1240	181	7	64 (5)	2.2	2
2003 OL Paper 2	42	935	106	9	65 (7)	3.2	3
2015 OL Paper 2	40	1051	117	9	86 (8)	3.3	3
2003 FL	32	530	73	7	23 (4)	1.6	0
2015 FL	43	1027	140	7	57 (6)	2.3	2

HL = Higher level; OL = Ordinary level; FL = Foundation level.

The analysis of the mathematics component of three randomly-selected TIMSS 2015 booklets shows that the average reading difficulty ranged from US Grade 1 to Grade 2, with relatively few difficult words (between 2% and 5%) (Table 6). Hence, the TIMSS mathematics test is somewhat easier in terms of readability than the Junior Certificate 2015 Higher level papers (which are written at a US Grade 4 level), and is closer to Junior Certificate Foundation level (both are about US Grade 2). However, vocabulary in TIMSS tends to be more technical than in the Junior Certificate Ordinary level papers, a feature not reflected in estimates of readability.

As noted in the introduction to this paper, King and Burge (2015) reported that the average readability of PISA clusters (students typically complete one or two of these, along with clusters in reading and/or science), ranged from Grades 8 to 11, with word counts ranging from 821 to 1193 words. Hence, compared with TIMSS and the Junior Certificate examinations at Higher, Ordinary and Foundation levels, the PISA mathematics test is written at a much higher level of difficulty. Furthermore, students taking two clusters of PISA mathematics (one hour of testing time) are asked to read about 2000 words, compared to 1200 words in TIMSS (for 45 minutes of testing), and 1050 in the 2015 Junior Certificate Ordinary level Paper 2 (for two hours of testing). Even at Junior Certificate Higher level in 2015, where students were asked to read 1335 words (Paper 1, over 2.5 hours) and 1620 words (Paper 2, over 2.5 hours), the reading load was proportionately lower than for one hour of PISA mathematics, and the readability level was considerably lower.

Table 6
Readability Measures for TIMSS 2015 Mathematics Booklets

Booklet	No. of Item Parts*	No. of words	No. of sentences	Avg. no. words per sentence	No. of complex words (%)	Flesch-Kincaid grade level	Avg. read-ability grade
Booklet X	29	1161	128	9	55 (5)	2.4	2
Booklet Y	32	1197	144	8	49 (4)	2.4	1
Booklet Z	31	1292	155	8	30 (2)	1.9	1

*Numbers of items: Booklet X – 28 items; Booklet Y – 28 items; Booklet Z – 26 items.

CONCLUSION

As the results of the analysis show, there were some changes to the content of the Junior Certificate mathematics examination between 2003 and 2015. The most salient difference was the increase in the number of items relating to Data and Chance (TIMSS) and Uncertainty (PISA), which is to be expected given the increased emphasis on Statistics and the introduction of Probability in the Project Maths syllabus. This change was observed at all examination levels. At Foundation level, there was a substantial reduction in Number/Quantity items, reflecting a move away from the previously large proportions of basic computation items and simple problems relating to concepts in Applied Measure such as Time and Money, in favour of other content areas such as Geometry. The PISA content classifications for 2003 were broadly consistent with those of Close and Oldham (2005), with some differences arising from the inclusion of a ‘Not covered’ category in the present study, as well as changes to how some Measurement items were classified.

Given the emphasis in Project Maths on problem-solving and depth of understanding, it might be expected that more examination items assessing higher-order cognitive processes and fewer assessing lower-order processes would be found on the Junior Certificate mathematics examination. Analysis using the TIMSS and PISA frameworks indicates that the former has occurred to some extent. For example, based on the TIMSS Cognitive Domains, there were small increases between 2003 and 2015 in the proportions of Reasoning items at Higher and Ordinary levels. At Foundation level, the proportion increased from zero to 12 percent. However, the proportion of Knowing items was similar at Higher level for both years and actually increased at Ordinary level, with nearly half of the 2015 Ordinary level items classified as Knowing. This suggests that, while the 2015 examination contained slightly more advanced items at all levels, there were still many items which drew on processes such as recall of concepts and performance of algorithmic procedures. It should also be noted that, even at Higher level, the proportion of Reasoning items in 2015 fell considerably short of the 25% target outlined in the TIMSS framework.

In terms of the PISA Competencies, the classifications in the current study were broadly in line with those of Close and Oldham (2005), with slightly more items classified as Connections at Higher and Ordinary levels in the present analysis. The most notable finding was the absence of Reflection items in all but one of the Junior Certificate examinations. Only two of the 2015 Higher level items were in this category, with none at any other examination level. There were moderate reductions in Reproduction items across all levels with accompanying increases in Connections. However, at all levels, the vast majority of items remained in the Reproduction category. Notably, the proportions of Reproduction and Connections items at Ordinary and Foundation levels in 2015 were very similar. This does not necessarily indicate that the examinations were of similar difficulty, but rather that a similar proportion of the items required students to demonstrate knowledge of familiar and practised material. Furthermore, at Foundation level, the items classified as Connections were not necessarily particularly difficult, but did require students to link different representations of mathematical concepts, such as matching written and symbolic statements. It is important to highlight that the 2015 examinations differed from 2003 with the inclusion at Higher and Ordinary levels of items that asked students to explain or justify results and conclusions. These items account for some of the increases in higher-order skills detected in the analysis, reflecting the new learning outcomes in the Project Maths syllabus relating to synthesis and problem solving. Taken together, these findings suggest that, while there were undoubtedly some moves in 2015 towards assessing higher-order processes, the changes did not represent the shift in focus that might be expected from a reform as comprehensive as Project Maths. In addition, in terms of the distribution of cognitive processes, the Junior Certificate examination is more closely aligned with the TIMSS framework than with PISA.

As the analyses described in this paper show, there was an increase in the use of real-life contexts between the 2003 and 2015 Junior Certificate examinations, at all levels. There was a marked increase at Higher level, with more modest changes at Ordinary and Foundation levels. This perhaps reflects the aims of the previous and revised curricula for different learners. In the curriculum in place in 2003, the Higher level course focused on 'material that underlies academic mathematical studies' (DES/NCCA, 2000 p. 7), while the Ordinary and Foundation level courses were heavily concerned with practical topics, with some exposure to more abstract mathematics. As a result, the real-world focus of Project Maths could be expected to have a more noticeable effect on the Higher level examination. In addition, the new emphasis on Statistics and Probability had an impact on the proportion of items placed in context, as these content areas lend themselves to real-world situations.

As with the analysis of cognitive processes, the analysis of context indicated that the Junior Certificate examination is more in line with TIMSS than with PISA. This is the case in terms of the proportion of items placed in context (about half), but also in terms of the way the contexts are presented. In the PISA assessment, all mathematics items are placed in a real-world context, which can be rich and complex, often with a significant amount of accompanying text, but with consequences for the readability of the assessment. Answering the associated items usually requires some degree of 'mathematisation', which involves extracting the mathematical components from the context in order to solve the problem, followed by some interpretation to relate the results back to the original context. While many of the 2015 Junior Certificate items were classified as having an 'Authentic' context, these do not generally correspond to the PISA model. In some cases, this is because the Junior Certificate items relate to mathematical concepts that are inherently practical, such as tax credits or interest on savings. In general, the presentation of Junior Certificate contexts is more similar to

TIMSS; both are introduced with relatively little text. In addition, it is usually clear what is expected of students, in terms of the mathematics they are required to use. However, the 2015 examination papers do show evidence of efforts to make meaningful links with the real world in content areas that were previously assessed in mostly abstract ways. As an example, the 2015 Higher level examination included a question on representing mobile phone price plans as functions, with an item requiring students to interpret the y-intercept of each function to determine whether the companies charged a fixed monthly fee (see Higher level Paper 1, 2015, Q6).

The overall structure of the Junior Certificate examination was not an intended focus of this study, but during classification of the items, some observations on this were made. First, the number of main questions in the examinations increased between 2003 and 2015. The 2003 examination papers all had six main questions, while the 2015 papers had up to 14 (Higher level Papers 1 and 2). In terms of individual item parts, the Higher level examination increased from 73 items to 84 (both papers combined). At Foundation level, there was an increase from 32 to 43, while the number at Ordinary level was similar in both years (82 to 81). In 2003, the Junior Certificate mathematics questions were structured with a, b and c parts, usually with increasing difficulty. The c part was sometimes unrelated to preceding question parts and, especially at Higher level, could be considered to require a higher cognitive demand than parts a and b. In contrast, the 2015 examinations followed a slightly different format with the sub-parts of each main question usually related. Question parts usually increased in difficulty, but ‘scaffolding’ was often used. In this approach, students were led through a series of steps to help them to answer the more challenging later items in the question. This scaffolding was presumably intentional at Ordinary and Foundation levels, as the assessment section of the new syllabus explicitly states that students at these levels will receive ‘structured support’ throughout the examination (more so at Foundation level). However, there is evidence from the 2015 examinations that this also occurred at Higher level, though to a far lesser degree (see, for example, Paper 1 Q14 and Paper 2 Q13c).

It is difficult to comment on differences in student performance in the Junior Certificate examinations between 2003 and 2015 for a number of reasons. As outlined earlier, there have been some changes in the achieved grade levels between 2003 and 2015, but much of this is likely to be attributable to changes in the cohorts of students at each examination level. In addition, state examinations are not designed to measure trends as such, with different questions each year and associated, non-static, marking schemes. However, the Chief Examiner's report for mathematics in 2015 (SEC, 2015b) provides some insights into students' capabilities. One of the strongest conclusions in this report was the observed shortfall in basic algebra skills at Ordinary and Foundation levels. The Chief Examiner also noted some decline in the standard of algebraic manipulation at Higher level. Similar concerns were raised by Prendergast and Treacy (2015) in relation to students entering third-level education who had been taught algebra through the Project Maths syllabus. However, the Chief Examiner also noted a number of positive changes. These included an increased willingness among students to engage with less familiar problems, and an improvement in students' abilities to answer questions requiring written explanations. Candidates were also found to have used a range of methods to arrive at their answers. These findings are important in the context of the cognitive processes described in this study, suggesting that, as the new Project Maths approach becomes more embedded, there could be scope to further increase the proportions of items assessing higher-order skills (Reasoning in TIMSS and Connections and Reflections in PISA). However, it is worth noting that the most difficult item in the 2015 Higher level examination was one of the two Reflection items, which was answered satisfactorily by only a small minority of students (see Paper 1, 2015, Q14 d). This was the case despite the presence of 'scaffolding' throughout the preceding question parts.

While the outcomes of PISA 2015 may provide further insights into the effects of Project Maths on performance as all 15-year-old students in Ireland in that study had learned mathematics under Project Maths, interpretation of outcomes may be confounded by the transition to computer-based assessment. This would suggest that, at some future time, it might be worth re-administering aspects of the 2003 Junior Certificate examination to a representative sample of students. This would help identify areas in which performance has increased or declined since the implementation of Project Maths. Unfortunately, Ireland did not participate in TIMSS between 1995 and 2015 so there are limited trend data available. However, performance in TIMSS in 2015 provides a broad indication of how students who studied under Project Maths perform relative to their counterparts in other participating countries.

If the readability of mathematics texts is defined by the attributes of conventional readability formulae (i.e., word difficulty/length and sentence length), it can be concluded that the TIMSS mathematics test, which has an average readability level of US Grades 1-2, is written at about the same level of difficulty as the 2015 Junior Certificate Foundation level examination (US Grade 2), and is a little easier than the 2015 Junior Certificate Ordinary level examination (US Grade 2 on Paper 1 and US Grade 3 on Paper 2), and the 2015 Junior Certificate Higher level examination (US Grade 4 on both Papers 1 and 2). Moreover, in terms of reading load (which does not take account of the inherent complexity in diagrams and symbols), students taking the Junior Certificate examination encounter fewer words (relative to available time) than in TIMSS. PISA, on the other hand, requires students completing its mathematics clusters to read more text, written at a higher level of difficulty (US Grades 8-11 on average).

As noted in the introduction, teachers, in particular, expressed concern about the reading load and complexity of textbooks and examinations associated with Project Maths. The findings reported in this paper indicate that the reading load (number of words) in the Junior Certificate mathematics examination increased across all levels between 2003 and 2015, ranging from 77% (Higher level Paper 1) to 94% (Foundation level). Hence, students are required to read more text now (i.e., an increased reading load), compared with 2003. However, this has to be considered in light of the following:

- the readability of Junior Certificate papers, as measured by conventional readability formulae, has changed little since 2003;
- the vast majority of word problems are presented in mainly familiar practical contexts, making it relatively easy to identify the underlying mathematics (compared with, for example, PISA); and
- students are generally supported as they move through the questions by scaffolding.

PISA, on the other hand, requires students to read considerably more text than either TIMSS or the Junior Certificate mathematics examination and texts are written at a considerably higher readability level. This means that students taking PISA, and less-able readers in particular, must process quite complex text before they can access the underlying mathematics and attempt to solve a problem. This, in part, contributes to the relative difficulty of PISA. Although students in Ireland are consistently among the highest-performing PISA countries in reading literacy in terms of average performance (Perkins et al., 2013), the high reading load and the complexity of mathematics texts must affect the performance of students with relatively low levels of proficiency in reading (see, for example, Mullis, Martin & Foy, 2013). As PISA and TIMSS progress with the implementation of computer-based assessment, it is likely that some form of adaptive testing will be introduced. While this may not lead to changes in overall performance, it will at least ensure that these assessments will become more accessible to lower-achieving students, including those with reading difficulties.

The analyses presented here indicate that there were a number of changes in the Junior Certificate mathematics examinations between 2003 and 2015 at all levels. There were expected changes to content, reflecting, for example, the stronger emphasis on Statistics and Probability in the Project Maths syllabus. Some small increases in the proportion of items assessing higher-order skills, as defined by the TIMSS and PISA frameworks, were also observed. However, these were not of the scale that might be expected given the fundamental changes in approach which characterise the new curriculum. Concerns about the reading load of mathematics assessments since the advent of Project Maths are partially supported, in that word counts dramatically increased between 2003 and 2015. However, the actual difficulty of the texts that students encountered was relatively stable. Overall, the analyses suggest that the Junior Certificate mathematics examinations are more closely aligned with TIMSS than with PISA. This does not support the view expressed by Kirwan (2015) and others that PISA has had an undue influence on the new curriculum, at least not in terms of how it is assessed in the Junior Certificate. If anything, we would argue that there is scope for the inclusion of more items assessing higher-order mathematics processes at all levels of the Junior Certificate examination. We also see a need for some in-depth analysis of teaching methods in Junior Certificate mathematics classes, perhaps extending the work of Lyons et al. (2003), to ascertain the ways in which the Junior Certificate examination now impacts on teaching and learning. Surprisingly, few if any studies on the implementation of Project Maths have looked at this.

Finally, it should be noted that the current paper does not address issues raised in the literature in relation to Leaving Certificate mathematics. There is a clear need for research that systematically examines the effects of changes to the Leaving Certificate mathematics syllabus in the context of Project Maths, as well as research on how examination papers at that level have evolved in response to Project Maths and other systemic changes, such as the availability of bonus points for Leaving Certificate mathematics since 2012.

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APPENDIX

Table A1:
Agreement Rates for Initial Item Classification of Items by Process

Exam Paper Year	Exam Paper Level*	TIMSS Cognitive Domain % Agreement	PISA Competency Cluster % Agreement
2003	Higher Level	78.4	86.5
	Ordinary Level	80.5	93.9
	Foundation Level	75.0	96.9
2015	Higher Level	79.8	81.0
	Ordinary Level	82.7	95.1
	Foundation Level	74.4	86.0

*For Higher and Ordinary levels, Papers 1 and 2 were collapsed and an overall level of agreement calculated.

Table A2:
Log of Issues Arising from the Classification of Junior Certificate Mathematics Items

Classification dimension	Issue	Classification
Content	Set Theory not in TIMSS or PISA frameworks	TIMSS: 'Not covered' PISA: 'Change & Relationships'
	Trigonometry (except Pythagoras) not in either assessment e.g. trigonometric ratios	TIMSS: 'Not covered' PISA: 'Not covered'
	Algebra: Factorising and solving quadratic equations not in either assessment	TIMSS: Algebra PISA: Change & Relationships
	Measurement items: Quantity or Space & Shape in PISA	Space & Shape if some spatial reasoning involved. Quantity for straightforward application of measurement formulae.

Table A2 Continued

Classification dimension	Issue	Classification
Process	Solving simultaneous equations	Applying (as per TIMSS 2015 classifications)
	Solving quadratic equations; factorising expressions; dividing expressions	Applying (K. Cotter, TIMSS & PIRLS International Study Center, personal communication, September 22, 2016)
	Using measurement formulae: intra-mathematical (no context)	Knowing – Compute (K. Cotter, TIMSS & PIRLS International Study Center, personal communication, September 28, 2016)
	Applying measurement formulae to solve problems (real-life context)	Applying – Implement (K. Cotter, TIMSS & PIRLS International Study Center, personal communication, September 22, 2016)
	Application of co-ordinate geometry formulae e.g. midpoint, equation of line etc.	Applying (as per TIMSS 2015 classifications for midpoint item)
	Constructions	Applying (no sub-category in Knowing that covers drawing figures)