PISA in Classrooms:

Implications for the teaching and learning of mathematics in Ireland

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PREFACE

The Programme for International Student Assessment (PISA) is a study of the Organisation for Economic Cooperation and Development (OECD). The study, which runs every three years, measures the mathematical, science and reading literacy of 15-year-old students in over sixty countries. In each cycle, one subject area is designated as the main focus of the assessment. In 2012, the main focus was on mathematical literacy.

This report, which is intended for teachers of mathematics in post-primary schools, focuses on the outcomes for students in Ireland and their implications for teaching and learning. The report is divided into eight chapters. Chapter 1 provides a brief overview of PISA and the performance of students in Ireland in mathematics. Chapter 2 considers the performance of girls in Ireland in mathematical literacy and how it might be improved, while Chapter 3 explores ways to reduce anxiety related to mathematics. Chapter 4 examines how lower-achieving students in Ireland could be supported further and Chapter 5 looks at how performance among higher-achieving students could be improved. Chapter 6 considers ways to enhance opportunities to learn mathematics and Chapter 7 discusses broadening the use of Information and Communication Technologies (ICTs) in mathematics classes. Finally, Chapter 8 presents some examples of mathematics items that formed part of the PISA mathematics assessment in 2012.

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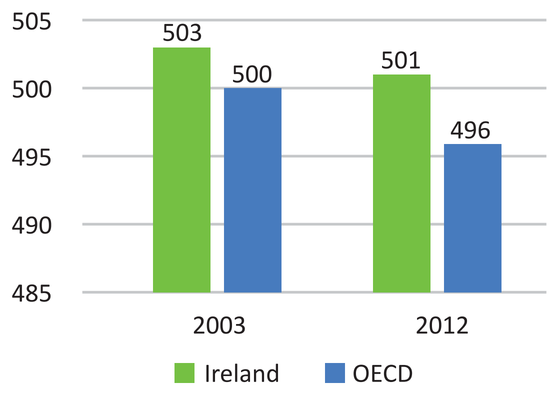
1. WHAT IS PISA MATHEMATICS AND HOW DID STUDENTS IN IRELAND DO?

PISA (the Programme for International Student Assessment) is a study of the Organisation for Economic Cooperation and Development (OECD) that measures 15-year-old students' reading, science and mathematical literacy in over sixty countries. The study runs every three years, with one subject area designated as the main focus of the assessment in each cycle. In 2012, the main focus was on mathematical literacy. PISA is designed to assess the extent to which students can apply the skills and knowledge they learn in school to real-life situations and, as such, is not directly linked to the school curriculum.

PISA selects students from across different grade levels. In Ireland, about 60% of students who participated in the study were in Third Year, almost 25% were in Transition Year and the remainder were in Second and Fifth Year.

On average, students in Ireland performed significantly better than their OECD peers on mathematics (see Figure 1). Of all the 65 countries that participated in PISA in 2012, 16 had an average mathematics score that was significantly higher than Ireland's. Ireland's average performance in mathematics in 2012 has not changed considerably since 2003, the last time mathematics was the main focus of PISA.

Figure 1: Average mathematics performance in Ireland and across OECD countries, in 2003 and 2012



PISA mathematical content areas

Change & Relationships involves understanding types of change and recognising when they occur in order to use suitable mathematical models to describe and predict change.

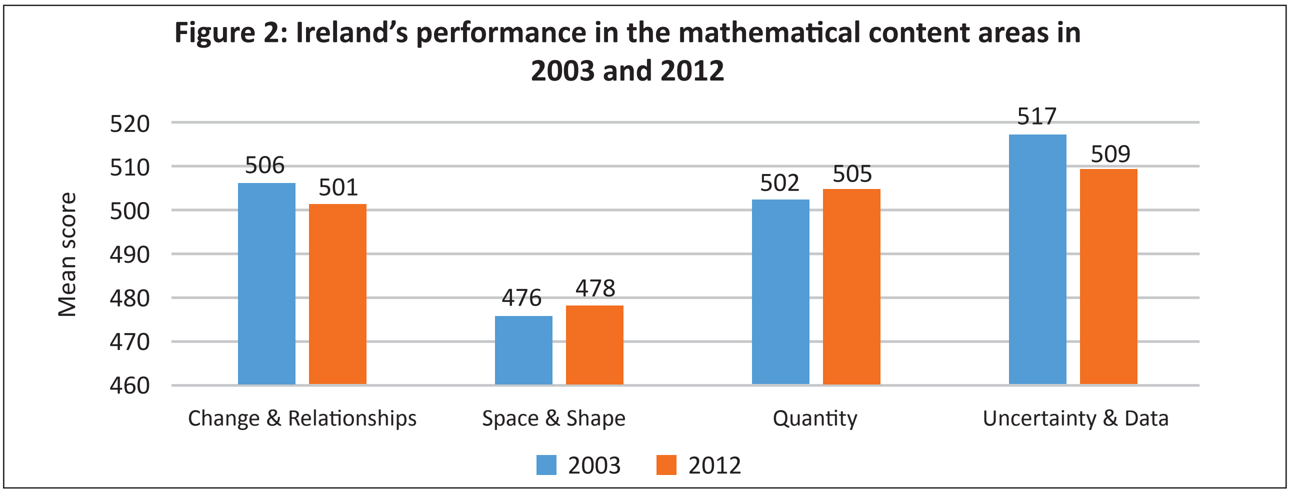
Space & Shape involves understanding perspective, transforming shapes with and without technology, interpreting views of three-dimensional scenes from various perspectives, creating and reading maps and constructing representations of shapes. This content area draws on geometry, spatial visualisation, measurement and algebra.

Quantity involves understanding measurements, counts, magnitudes, units, indicators, relative size, and numerical trends and patterns, and employing number sense, multiple representations of numbers, mental calculation, estimation, and assessment of reasonableness of results.

Uncertainty & Data involves knowledge of variation in processes, uncertainty and error in measurement, chance and descriptive statistics.

However, while Ireland's average mathematics performance did not differ from the OECD average in 2003, it is significantly above it in 2012, mainly owing to a decrease in the OECD average between 2003 and 2012.

PISA measures student achievement in four mathematical content areas (see inset). Ireland had significantly higher mean scores in the Change & Relationships, Quantity and Uncertainty & Data content areas compared to the corresponding OECD average scores; however, Ireland performed significantly less well in the Space & Shape content area. There has been little change in the performance of students in Ireland in the four mathematical content areas between 2003 and 2012, with the exception of Uncertainty & Data, where performance declined significantly by almost 9 points (see Figure 2). This decline was particularly marked among higher-achieving students (a decline of 13 points).



AREAS OF RELATIVE STRENGTH FOR BOYS AND GIRLS IN IRELAND

In Ireland boys significantly outperformed girls on the overall mathematics scale (509 compared to 494) and in each of the mathematical content areas. Both boys and girls in Ireland performed best in the Uncertainty & Data content area and least well in the Space & Shape content area (Table 1).

Table 1: The performance of boys and girls in Ireland across mathematical content areas, relative to the OECD average

	Above OECD average	Similar to OECD average	Below OECD average
Boys	*Change & Relationship* *Quantity* *Uncertainty & Data*	*Space & Shape*	
Girls	*Change & Relationship* *Quantity* *Uncertainty & Data*		*Space & Shape*

Although students in Ireland performed well in mathematics compared to the average across OECD countries, there are still notable areas where improvements can be made. Six key themes emerged from the PISA 2012 mathematics findings that are considered to be particularly important for improving performance among students in Ireland. These themes are:

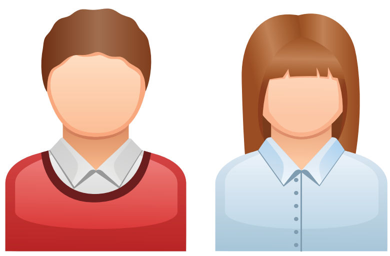
1. the relatively poor performance of girls,
2. high levels of mathematics anxiety among students,
3. addressing the needs of lower performing students,
4. the relative underperformance of higher-achieving students,
5. opportunity to learn mathematics and
6. the use of Information and Communication Technologies (ICTs) in the teaching and learning of mathematics.

Each of these themes is addressed in the remainder of this report and associated recommendations for classroom practice are provided. It should be noted that the six themes are inter-related (for example, gender and anxiety) and, as such, recommendations may apply across themes. It should also be noted that the vast majority of students who took part in PISA 2012 did not have any formal exposure to the new mathematics curriculum introduced under the Project Maths initiative and therefore findings and recommendations should be considered in that context.

Some examples of items used in the PISA mathematics assessment are provided at the end of this report.

2. IMPROVING GIRLS' PERFORMANCE IN MATHEMATICS

PISA tells us that, on average, boys achieved better on mathematical literacy questions than girls in Ireland and that this advantage was also apparent when achievement was examined across each of the different mathematical content areas (OECD, 2013). Moreover, the overall gap in performance between boys and girls was larger in Ireland than on average across OECD countries (by almost 5 points). When the distribution of mathematics achievement is examined, it can be seen that there were more lower-achieving girls, and fewer higher-achieving girls, than boys, in Ireland.



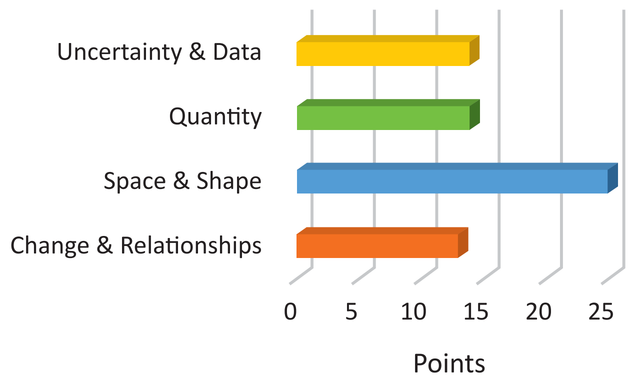
Ireland had a larger gender gap in mathematics (15 points) than on average across OECD countries (11 points)

These findings are in contrast to performance on Junior Certificate mathematics, where girls are more likely than boys to receive higher grades across syllabus levels, with the exception of an A grade at Higher Level (State Examinations Commission, 2015). This suggests that it is on the more complex, higher-level tasks where girls fall down and PISA provides some evidence for this. The performance of boys at the 90th percentile was higher than that of girls in Ireland, and the difference (19 points) was larger than the difference between boys and girls at the 10th percentile (8 points; OECD, 2015).

As the percentage of girls taking Higher Level mathematics tends to be slightly greater than boys (State Examinations Commission, 2015), it is unlikely that exposure to mathematical content is a problem. However, it is possible that girls are less likely to engage with more complex tasks, such as those that appear in PISA, for a variety of reasons, including lower self-efficacy and greater anxiety related to mathematics (see page 5).

Of particular concern is the relatively poorer average performance of girls in Ireland on the Space & Shape content area. Girls had an average score that was almost 25 points lower than boys (see Figure 3). Also, the average performance of Irish girls on the Space & Shape content area was below the average performance for girls across OECD countries. While the types of items that measure Space & Shape in PISA are likely to be more familiar to students who have studied under the new mathematics curriculum, these types of items are not represented in the previous Junior Certificate Geometry strand at any syllabus level (Close & Oldham, 2005) and most Irish students who participated in PISA 2012 would not have had any formal exposure to the new mathematics syllabus. Boys may have more experience developing the skills required for the practical application of spatial relations through formal and informal experiences in other subject areas, which could contribute to their relative advantage.

Figure 3: Differences between boys and girls across mathematical content areas in Ireland



Items on PISA Space & Shape involve a range of activities such as understanding perspective, creating and reading maps, transforming shapes with and without technology, interpreting views of three-dimensional scenes from various perspectives, and constructing representations of shapes. Geometry is viewed by PISA as being central to Space & Shape, although the formal system with theorems and proofs, which is a major element of the Irish mathematics syllabus, is almost unrepresented in PISA. Aspects of other content areas such as spatial visualisation, measurement, number and algebra are also drawn on. The manipulation and interpretation of shapes in settings such as dynamic geometry software and Global Position System (GPS) tools are included in this area, though not necessarily represented in current PISA Space & Shape items.

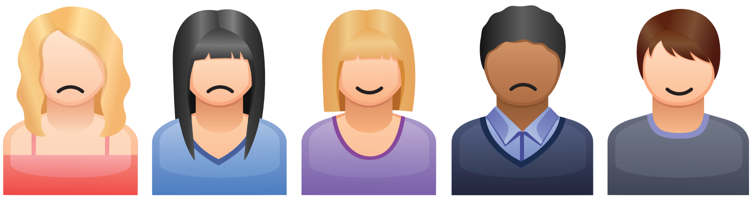
Girls' relative underperformance on PISA is likely to be due to a number of reasons, including a noted weakness among girls on Space & Shape items, a greater focus on higher-level competencies in PISA, girls' confidence in their own abilities in mathematics and a differing approach among girls to assessments such as PISA compared to other 'high-stake' assessments such as state examinations (Close & Shiel, 2009). Some practical suggestions for improving understanding of mathematics among girls in Ireland include:

- Encouraging girls, especially higher-achieving girls, to engage with more complex, higher-level tasks and to explore solving problems in novel ways.
- Making use of software linked to spatial reasoning, geometry and functions, by both teachers and students, in mathematics lessons to encourage the development of spatial reasoning skills among girls.
- Engaging students in more technical subjects, perhaps through short courses at Transition Year, to better develop girls' spatial relations skills.
- Availing of opportunities for integrating concepts associated with Space & Shape in subjects like geography (e.g., location of cities in relation to one another, map reading, orientation, grid references, GPS readings, latitude and longitude, time zones), history (location, buildings, archaeology, sense of time and space), science (shape in natural phenomena, properties of particles), and literature (timescales, direction) (see Fox & Surtees, 2010).

3. REDUCING LEVELS OF MATHEMATICAL ANXIETY

As well as completing the tests, PISA 2012 students were asked about their mathematics self-efficacy (their belief that they can solve various mathematical tasks), their mathematics self-concept (their belief in how well they are doing in mathematics), and their anxiety about mathematics (see Table 2). Ireland's average self-efficacy and self-concept scores were about the same as the averages of students across OECD countries, while the average level of mathematics anxiety was significantly higher, indicating that mathematics anxiety is a significant issue for Irish students (OECD, 2013). The gap in performance between boys and girls in Ireland (15 points) was more than halved (to 7 points), when anxiety was accounted for (OECD, 2015).

On average, students in Ireland who had higher levels of mathematics anxiety tended to have lower mathematics achievement. However, this is likely to be a complex relationship where student performance might impact on anxiety and vice versa. Nevertheless, reducing levels of mathematical anxiety, especially among girls, who have significantly higher levels of mathematical anxiety than boys in Ireland, could improve engagement with mathematics, a key objective of the new mathematics curriculum.



In Ireland,
2 in 3 GIRLS vs 1 in 2 BOYS
report worrying that they will get poor grades in mathematics

Baroody and Costlick (1998) describe how unreasonable beliefs can lead to anxiety, anxiety can then lead to protective behaviour such as avoidance, and protective behaviour, in turn, can reinforce unreasonable beliefs. Students who experience anxiety related to mathematics generally avoid mathematics, mathematics courses and career paths that require mastery of mathematical skills (OECD, 2015). Rossnan (2006) suggests that a focus on memorising mathematical concepts rather than working through problems and understanding the reason behind mathematics concepts can also contribute to mathematics anxiety.

Allowing time for routine work can help reduce mathematics anxiety, but students should also be encouraged to take risks and use higher-order processes by engaging with non-routine problems. Teachers should use their professional expertise and knowledge of individual students to try to balance these conflicting demands.

Table 2: Percentages of boys and girls in Ireland who agreed or strongly agreed with various statements about mathematics anxiety

	Ireland		OECD average	
Percentage who agreed or strongly agreed that…	Boys	Girls	Boys	Girls
I often worry it will be difficult for me in mathematics class	64%	76%	54%	65%
I worry I will get poor grades in mathematics	55%	69%	56%	67%
I get very tense when I have to do mathematics homework	32%	40%	30%	35%
I get very nervous doing mathematics problems	24%	36%	27%	34%
I feel helpless when doing a mathematics problem	22%	34%	25%	35%

In addressing mathematics anxiety, particular attention should be paid to girls. Some ways to reduce anxiety related to mathematics, suggested by Cruikshank and Sheffield (1992) and Ashman (2015) include:

- Modelling a positive attitude towards mathematics by displaying the use of mathematics in everyday life and careers.
- Providing activities that students can complete successfully and establishing short-term and attainable goals for students.
- Having frequent, short-duration, low-stakes tests that emphasise routine competence as well as non-routine problems.
- Using easy but unfamiliar problems to familiarise students with problem solving and to build confidence.
- Developing alternatives to written tests, such as student work folios, journals and observation.

4. SUPPORTING LOWER-ACHIEVING STUDENTS

PISA refers to lower-achieving students as those who scored at or below Level 1 on the PISA overall mathematics proficiency scale. Seventeen percent of students in Ireland were considered to be lower-performing students, compared with an OECD average of 23%. These students are viewed as having insufficient mathematical skills to benefit from future learning opportunities, or to apply mathematics in real life situations. Of the 17% of lower achievers in mathematics in Ireland, less than half were also lower achievers in PISA reading and science.

1 in 6 students in Ireland perform poorly in mathematics

Key skills that students performing at Level 1 are likely to demonstrate are described in Table 3, as well as the somewhat more advanced skills demonstrated by students performing at the next highest level (Level 2). PISA places both student performance and the difficulty levels of items on the same underlying scale (i.e, items that students performing at Level 1 are likely to succeed on are labelled as Level 1 items). The descriptions in the table are based on the types of skills that are needed to answer items at each of these levels correctly. They show the progress between the two levels, and should be interpreted with reference to sample items presented at the end of this report.

Table 3: Descriptions of the types of tasks that students can typically perform at PISA levels 1 and 2

What students can typically do at PISA Level 1 (below baseline level)	What students can typically do at PISA Level 2 (baseline; minimal level for further study/effective use of mathematics in real life)
Answer questions involving familiar contexts where all the relevant information is present, and the questions are clearly defined.	Interpret and recognise situations in contexts that require direct inferences.
Identify information and carry out routine procedures according to direct instructions in explicit situations.	Extract relevant information from a single source and make use of a single representational mode.
Perform actions that are almost always obvious and follow immediately from a given stimulus.	Employ basic algorithms, formulae, procedures or conventions to solve problems.

Source: (OECD, 2013)

Risk factors associated with lower achievement on PISA mathematics include:

- *Gender* – In Ireland, more girls (19%) than boys (15%) performed at or below Level 1 on PISA mathematics. On Space and Shape items, more girls (31%) than boys (23%) also performed at this level.

- *Socio-economic disadvantage* – In Ireland, 30% of the most disadvantaged students (those in the bottom quartile of the PISA measure of economic, social and cultural status) were identified as lower achievers in mathematics, compared with 5% of the top quartile.

- *Absence from school* – 7% of lower achievers skipped at least one day of school in the two weeks prior to the PISA 2012 assessment, compared with 4% of those who performed above Level 2.

- *Perseverance and attitude* – Lower-achieving students had significantly lower levels of general perseverance, lower interest in mathematics, and lower self-efficacy (confidence) in their mathematical ability, compared with students who were above Level 1.

In addition to individual student factors, PISA identified a number of school- and classroom-level factors associated with lower achievement in mathematics:

- *School-level disadvantage* – Students in Ireland attending the most disadvantaged schools were four times more likely to have lower achievement in mathematics, compared with students in the least disadvantaged schools, even after accounting for individual differences in socioeconomic status.

- *Low teacher expectations* – Schools in Ireland whose principals reported that low expectations among teachers in general hindered student learning had more lower achievers in mathematics (29% of students), compared with schools whose principals indicated that low expectations did not hinder learning (15%).

- *After-school classes* – In Ireland, there was no difference in the proportion of lower-achieving students (15%) attending schools offering or not offering classes in mathematics outside school hours. On average across OECD countries, more lower-performing students in mathematics attended schools that offered after-school classes in mathematics than schools that did not.

- *Disciplinary climate in mathematics lessons* – Lower-achieving students in Ireland were more likely to report that students in their mathematics classes did not listen to what the teacher said, that the teacher had to wait for a long time for the class to settle down, and that students could not work well, compared with students at higher levels of achievement. However, in Ireland and across OECD countries, students at all levels of socioeconomic status showed greater levels of familiarity with mathematics concepts as disciplinary climate improved.

- *Immigrant students and other-language speakers* – In Ireland, there were no significant differences in the proportions of lower performers on PISA mathematics between students categorised as having an immigrant background (whether first or second generation), and those not identified in this way (18% and 17%, respectively), or between students who reported mainly speaking a language other than English and those who reported mainly speaking English (20% and 16%, respectively). On the other hand, on average across OECD countries, a much greater proportion of students with an immigrant background (36%) and students who mostly speak a language other than the language of the test (35%) were classified as lower-performers than those who do not have an immigrant background and mostly speak the language they were tested in (both 21%).

Where a number of risk factors occur in combination (for example, individual and school-level socioeconomic status), there is often a greater risk of lower achievement, compared with the risk associated with individual factors.

A number of strategies have been identified for tackling lower achievement in mathematics by the OECD (2016a, b) and by others (e.g., Burge & Sizmur, 2015; Perkins & Shiel, 2016). These relate to the schools/mathematics departments and to teachers.

For schools and mathematics departments:

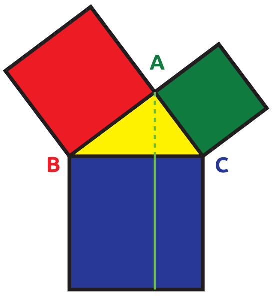
- Establishing teacher learning communities and lesson study groups in the school so that teachers can observe other teachers and share practices that are effective. Data from PISA 2012 and the OECD Teachers' Survey (TALIS 2013) indicate that the more teachers collaborate with other teachers in the same school, the more likely they are to use cognitive activation strategies (see page 9) in their classrooms.
- Providing extracurricular opportunities, both mathematics-related and recreational, after school hours.
- Ensuring that pupil engagement in mathematics has a strong focus at school and departmental levels by devising strategies to raise student engagement, and monitoring the effectiveness of engagement strategies in mathematics classes.
- Where students are grouped across or within mathematics classes, ensuring that there is timely movement between groups, in line with assessed performance.
- Implementing a policy of early diagnosis and intervention for lower-achieving students.
- Recognising that addressing lower achievement in mathematics requires a combination of short-term changes and long-term planning, and plan accordingly.

For teachers:

- Setting high expectations for all students, including lower-performing students.
- Allocating time, when introducing a new topic in mathematics, to explore students' understanding of the topic.
- Raising student engagement in mathematics by pointing out the relevance of topics to students' own lives.
- Raising students' cognitive engagement in mathematics by allowing students to decide on their own procedures when solving problems, assigning problems that can be solved in different ways, presenting problems in different contexts, giving problems with no immediate solution, and asking students to explain how they solved a problem.
- Providing lower-achieving students with opportunities to solve more complex problems so that they develop flexibility in applying what they have learned, rather than simply memorising routines and definitions.
- Ensuring that the learning environment (disciplinary climate) in mathematics classes is conducive to learning mathematics, and that requirements are clearly understood by students.
- Ensuring that students have opportunities to discuss and reflect on their learning, and to share their insights with peers (e.g., promote co-operative learning).
- Providing extra help and support to students who need it most.
- Raising the confidence and mathematical self-concept of lower-achieving students by promoting successful learning and providing appropriate feedback.

5. IMPROVING PERFORMANCE AMONG HIGHER-ACHIEVING STUDENTS

On average, students in Ireland performed significantly better on mathematics than their OECD peers (OECD, 2013). However, Ireland's above average score in mathematics is due to the comparatively good performance of lower-achieving students rather than strong performance across all ability levels. There is evidence that higher-achieving students in Ireland underperformed relative to their counterparts in other countries. The proportion of students in Ireland who were able to answer the most difficult PISA questions (those at PISA levels 5 and 6) was significantly below the corresponding OECD average (11% and 13%, respectively).



High achieving students in Ireland do relatively poorly on PISA mathematics

The particular areas in which higher-achieving students in Ireland underperformed are the Change & Relationships and Space & Shape mathematical content areas. It is also a concern that there was a drop in the performance of higher-achieving students in the Uncertainty & Data content area since 2003, though the assessment framework was the same on both occasions.

Cognitive activation, which is about teaching students strategies they can call upon when solving mathematical problems, was also measured in PISA. Overall, teachers' use of cognitive activation strategies (based on students' reports) was high in Ireland (see Table 4) but there was one strategy in particular where use in Ireland was below the OECD average: asking students to use their own procedures for solving complex problems (OECD, 2015).

Table 4: Frequency of cognitive activation strategies in mathematics lessons

Percentage of students who agreed or strongly agreed that ...	Ireland	OECD
The teacher asks us to explain how we have solved a problem	79%	70%
The teacher helps us to learn from mistakes we have made	72%	60%
The teacher asks questions that make us reflect on the problem	71%	59%
The teacher presents problems that require students to apply what they have learned to new contexts	68%	62%
The teacher gives problems that require us to think for an extended time	63%	53%
The teacher gives problems that can be solved in several different ways	59%	60%
The teacher presents problems in different contexts so that students know whether they have understood the concepts	59%	59%
The teacher presents problems for which there is no immediately obvious method of solution	50%	47%
The teacher asks us to decide on our own procedures for solving complex problems	31%	42%

Table 5 provides a list of the key skills that the highest achieving students in PISA (i.e., those at PISA levels 5 and 6) are likely to be able to demonstrate and that students performing below these levels might aspire to. These descriptions should be interpreted with reference to sample items presented at the end of this report.

Table 5: Descriptions of the types of tasks that students can typically perform at PISA levels 5 and 6

What students can typically do at PISA Level 5	What students can typically do at PISA Level 6
Develop and work with models of complex situations, including identifying constraints and specifying assumptions.	Conceptualise, generalise and use information based on investigations and modelling of complex problem situations.
Select, compare and evaluate appropriate problem-solving strategies for dealing with complex problems related to these models.	Use knowledge in relatively non-standard contexts and link different information sources and representations and move flexibly among them.
Work strategically using broad, well-developed thinking and reasoning skills, appropriate linked representations, symbolic and formal characterisations and insights pertaining to these situations.	Apply insight and understanding, along with mastery of symbolic and formal mathematical operations and relationships, to develop new approaches and strategies for addressing novel situations.
Begin to reflect on, as well as formulate and communicate their interpretations and reasoning.	Reflect on actions as well as formulate and precisely communicate their actions and reflections regarding their findings, interpretations and arguments and explain why they are applied to the original situation.

Source: (OECD, 2013)

Some strategies which may assist higher-achieving students include:

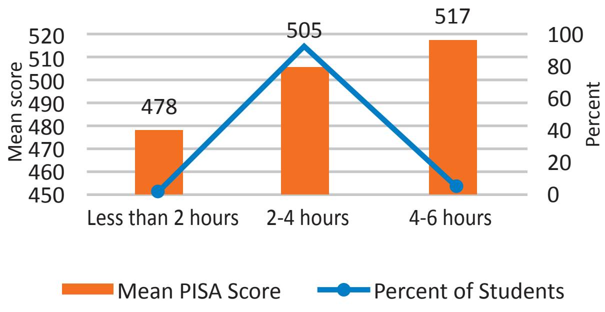
- Providing opportunities for higher-achieving students to engage with problems in novel contexts and to explore different solutions to problems, including through the use of technology. Students could be encouraged to design activities for Maths Week and to participate in activities such as the Young Scientist and Technology Exhibition, IMTA competitions and the Irish Mathematics Olympiad, as well as other enrichment activities.
- Encouraging students to engage with problems in new ways and to participate in more self-directed learning by making use of enrichment resources such as “Nrich” or “Numberphile”. Transition Year could also be used as an opportunity to develop short courses which aim to develop such skills.
- Encouraging students to engage with interactive platforms such as GeoGebra to promote engagement in Space & Shape tasks, such as rotation, which may broaden their understanding of the current curriculum.
- Encouraging students to talk and be more reflective about their mathematical thinking. Such behaviour could be promoted through the use of a more dialogical pedagogy in mathematics classrooms and by arranging students to work together in small groups to solve complex problems.
- Increasing engagement in online initiatives such as LearnStorm (http://www.learnstorm2016.org).

6. ENHANCING OPPORTUNITY TO LEARN MATHEMATICS

As well as describing 15-year olds' performance on mathematics and their attitudes to mathematics, PISA 2012 examined students' opportunity to learn (OTL) mathematics, based on students' own reports. Key measures of OTL included instructional time allocated to mathematics, students' reported familiarity with mathematical terms, frequency of engagement with particular mathematics tasks and problems, and frequency of working in groups in mathematics lessons.

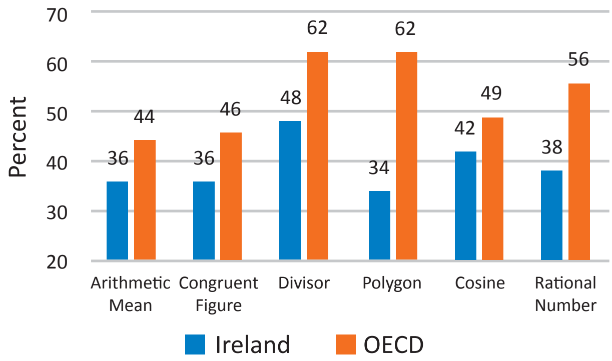
Instructional time. In Ireland, students reported spending an average of 189 minutes per week in mathematics classes, compared with an OECD average of 198 minutes. While average weekly time increased on average across OECD countries between 2003 and 2012, by 13 minutes, it remained unchanged in Ireland. One factor impacting on average instructional time in Ireland is Transition Year. Fifteen-year-olds in Transition Year in Ireland (25% of the PISA sample) reported attending mathematics classes for 160 minutes per week, compared to 195 minutes for students in Second/ Third years, and 219 minutes for Fifth years. In Ireland, and on average across OECD countries, students who spent more time in mathematics classes had higher average achievement (see Figure 4).

Figure 4: Association between instructional time per week and mathematics performance in Ireland

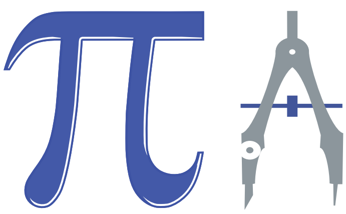


Familiarity with mathematical terms. Students in PISA 2012 indicated their familiarity with each of 13 mathematical terms (referred to as concepts by the OECD) on a scale ranging from 'know and understand the concept well' to 'never heard of the concept'. The list included exponential functions, vectors, polygons, congruent figures, arithmetic means, divisors, complex numbers, and probability. Fewer students in Ireland than on average across OECD countries indicated that they were familiar with each of these mathematical terms, with the exception of quadratic functions. Figure 5 compares the percentages of students in Ireland and on average across OECD countries reporting that they know selected terms well or have heard of them often. Across most terms (with the exception of vectors), students in Ireland with higher levels of socioeconomic status were more familiar with target terms than students of lower socioeconomic status (OECD, 2016b). The OECD describes familiarity with mathematics concepts as a measure that captures the cumulative opportunity to learn mathematical content over a student's career. However, it also recognises that, while familiarity with mathematical concepts is important, additional exposure to mathematical concepts (terms) in and of itself is not enough. Students also need extensive exposure to problems that 'stimulate their reasoning abilities and promote conceptual understanding, creativity, and problem-solving skills' (OECD, 2016b, p. 3).

Figure 5: Percent of students who know term well or have heard of it often in Ireland and OECD



Students in Ireland were less familiar with key mathematics concepts than on average across OECD countries



Contextualised mathematics problems – e.g., Compare two formulas for the recommended maximum heart rate based on age [200-age; 208-0.7(age)] and identify the point at which the second one leads to an increase in maximum recommended rate.

Procedural mathematics tasks – e.g., Solve $2x + 3 = 7$; find the volume of a box with sides 4m, 5m and 6m.

Pure mathematics problems – e.g., Determine the height of a pyramid using a geometrical theorem; if n is any number, investigate if $(n + 1)^2$ is a prime number.

Experience with contextualised, procedural and pure mathematics tasks and problems. In PISA 2012, students were asked to indicate how often they encountered different types of mathematics tasks in class or on tests (see panel). The data pointed to a relative over-emphasis on procedural tasks in Ireland (77% reported frequent experience with these problems, compared with an OECD average of 68%). This is potentially problematic if students do not understand the concepts underlying the procedures they perform, but, instead, routinely apply computational routines, with limited flexibility. Fewer students in Ireland (26%) than on average across OECD countries (34%) encountered pure mathematics problems frequently. This is a concern to the extent that large proportions of students in high-scoring countries such as Korea (36%) and Japan (48%) report that they encountered such problems more frequently. More students in Ireland (27%) encountered contextualised problems frequently, compared with the corresponding OECD average (21%).

In general, across OECD countries, more frequent exposure to particular problem types was associated with higher overall performance on PISA mathematics. However, in Ireland, students who solved contextualised problems frequently performed significantly less well than students who rarely or never did so. This suggests that, prior to full implementation of the new mathematics curriculum in all schools, lower-achieving students were more likely to be asked to solve contextualised problems, compared with higher-achieving students.

Grouping students within mathematics classes. In general, Ireland does well on measures of the distribution of students' mathematics skills and knowledge across schools, with schools in Ireland being more similar to one another. For example, differences between schools in students' familiarity with mathematical concepts are among the lowest across OECD countries. However, there is evidence that students in Ireland are streamed for mathematics classes to a greater degree than in other OECD countries, with 99% grouped by ability compared to an OECD average of 77%. In Ireland, 12% of students attended schools where they were grouped for mathematics instruction from the beginning of First Year '(their mean PISA mathematics score is 467 points)', 77% were grouped from the beginning of Second Year (501 points) and 10% were grouped from the beginning of Third Year (503 points). The remainder were grouped at other times.

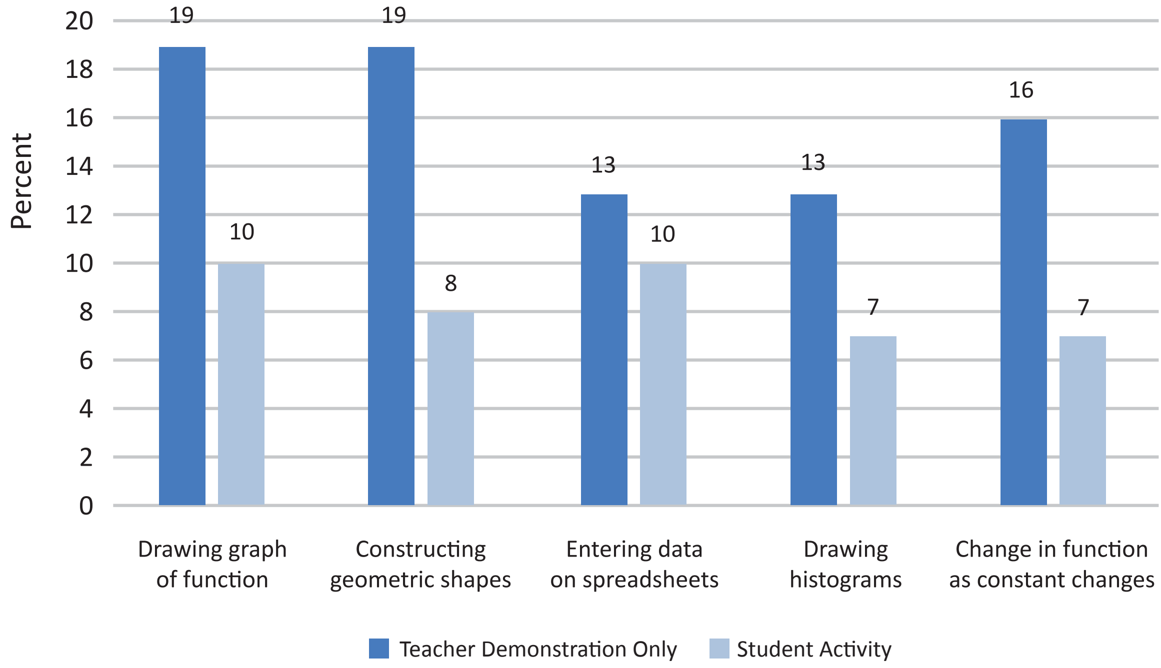
A number of strategies can be implemented in schools and classrooms to enhance the opportunities that all students have to learn mathematics. These include:

- Ensuring that adequate time is allocated to the teaching of mathematics, especially in Transition Year.
- Postponing implementation of streaming for mathematics for as long as possible.
- Where streaming is applied, flexible assignment to mathematics classes (syllabus levels) should be implemented so that students who make rapid progress can be promoted to more challenging mathematics classes.
- Establishing temporary, mixed-ability groups within mathematics classes so that students of differing ability levels can learn from and support one another.
- Ensuring that, where students are assigned procedural mathematics tasks, they fully understand the underlying concepts. Cognitive activation strategies such as encouraging students to reflect on problems, asking them to explain their answers, and supporting them in learning from their mistakes are especially relevant.
- Providing students at all levels of ability with contextualised problems, including problems that require them to apply what they learned in new contexts and understand its relevance.
- Ensuring that students at all ability levels have frequent opportunities to solve pure mathematics problems at an appropriate level of challenge, whether individually or in small groups.
- Encouraging students to participate in mathematics activities outside the formal classroom, such as Maths Week, maths trails and "Maths Eyes".

7. BROADENING USE OF ICTS IN MATHEMATICS CLASSES

Transition towards computer-based testing. In addition to completing a paper-based test in mathematics in PISA 2012, students in 32 countries, including Ireland, completed a computer-based test. Whereas students in Ireland performed at a level that was significantly above the OECD average on the paper-based measure, they performed at a level that was not significantly different from the OECD average on the computer-based measure. About one half of participating countries (including Ireland) performed better on paper-based than on computer-based mathematics. From PISA 2015 onwards, PISA will be administered in most OECD countries on computer-based format. Although it may be some time before state examinations in mathematics will be offered on computer, there are potential benefits to extending the use of computers in mathematics classes in terms of deepening students' understanding of key concepts and processes. Indeed, the new syllabus introduced under the Project Maths initiative (NCCA, 2013) anticipates that computer software will be used to enhance learning in all strands, including geometry (where dynamic software is recommended). The use of graphing technologies is especially emphasised.

Figure 6: Percentages of students who observed their teachers demonstrating various procedures using computers, and percentages who implemented the procedures



Use of computers in mathematics classes. As part of PISA 2012, students completed a number of questionnaire items about their use of ICTs in mathematics classes. While students in Ireland lagged well behind their counterparts in countries such as Denmark and Norway in terms of ICT usage, so did a number of high-performing countries such as Finland, Korea and Japan. Figure 6 shows the percentages of students in Ireland whose teachers demonstrated various procedures using computers in mathematics classes, and the percentages of students who implemented the procedures themselves. While about one-fifth of students observed teachers using computers to demonstrate procedures, fewer than one-tenth actually implemented procedures themselves. This probably reflects lack of access to computers by students in mathematics classes.

Promoting greater engagement with ICTs. It is recognised that successful deployment of ICTs in mathematics classes is contingent on a range of factors, including Internet speed, access to computing devices, teacher development, and relevance of software to performance on examinations. With implementation of the Digital Strategy for Schools 2011-2020 (DES, 2015), some of these challenges will be addressed over time. However, there is a need to broaden the use of computers in mathematics lessons in the short term.

1 in 5 students in Ireland observed their teacher demonstrate mathematics procedures on a computer, while 1 in 10 implemented these procedures themselves

Current progress on implementing ICTs into teaching and learning mathematics can be built on by:

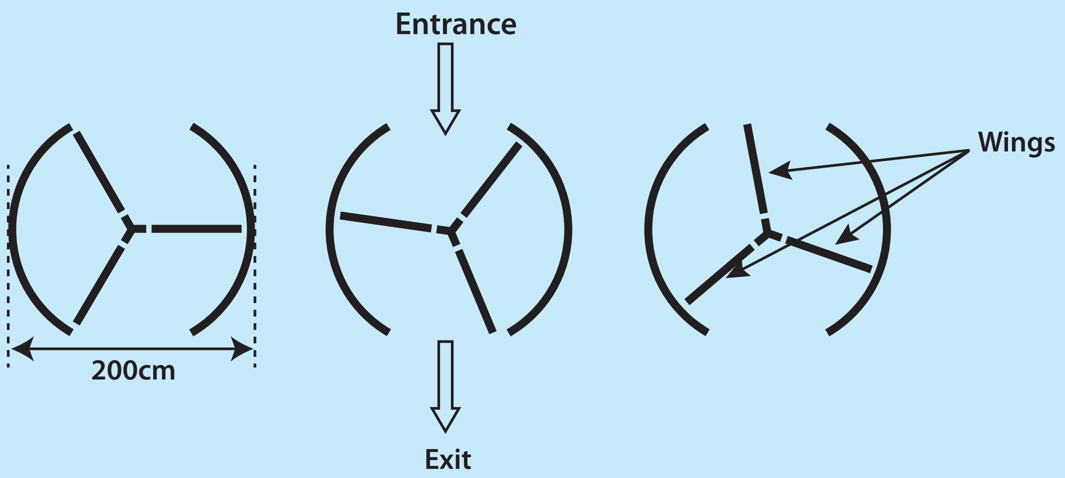
- Ensuring that, in addition to observing teacher-led demonstrations of mathematics processes using ICTs, students have an opportunity to use ICTs on an ongoing basis in their mathematics classes, whether in classrooms or in computer rooms.
- Using technologies in mathematics classes to support students in acquiring not only basic procedures but also higher-level processes.
- Ensuring that, over time, students are exposed to a range of technologies in mathematics classes including:
    - Dynamic geometry software
    - Software to enhance statistical reasoning
    - Internet applications, including collaborative tasks
    - Computer algebra systems
    - Graphics calculators (which can be used across strands)
- Maintaining a balance and appropriate interaction between traditional approaches where students solve problems by hand, and computer-based applications, where the focus shifts to more interpretative activity, including exploration of mathematical ideas.
- Using computers to offer enrichment in areas such as space and shape, which may be underemphasised in current syllabi, but which are nonetheless important for students' mathematical development across a range of areas.

8. WHAT ARE PISA MATHEMATICS ITEMS LIKE?

Some examples of items from the PISA mathematics test are included on the next few pages. The items presented represent a range of content areas and difficulty levels. Further examples of items from the PISA tests can be found at http://www.oecd.org/pisa/pisaproducts/pisa-test-questions.htm.

Below and overleaf is an example of a mathematics unit, called *Revolving Door*, with two questions that fall within the Space & Shape content area. The first question is considered to be a medium difficulty item (Level 3) and could be interpreted as asking about the angle formed by one-third of a circle. Although the presentation of this item may be unfamiliar to students in Ireland, they are likely to be familiar with the concept. Students in Ireland performed relatively well on this item, with 63% answering it correctly compared to 58% on average across OECD countries.

A revolving door includes three wings which rotate within a circular-shaped space. The inside diameter of this space is 2 metres (200 centimetres). The three door wings divide the space into three equal sectors. The plan below shows the door wings in three different positions viewed from the top.

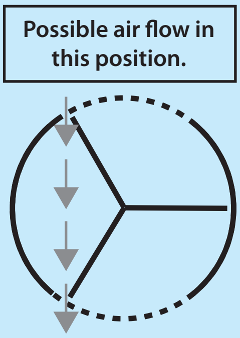


What is the size in degrees of the angle formed by two door wings?

Size of the angle: .... º

The second question is one of the most difficult PISA items (Level 6). The format of the second question may also be unfamiliar to students in Ireland, as the diagram shows the incorrect solution whereas demonstrations of the correct solution are more conventional in the Irish curricula. Students in Ireland, and across OECD countries, performed poorly on this item, with just 2.5% answering it correctly in Ireland and an average of 3.5% across OECD countries.

The two door **openings** (the dotted arcs in the diagram) are the same size. If these openings are too wide the revolving wings cannot provide a sealed space and air could then flow freely between the entrance and the exit, causing unwanted heat loss or gain. This is shown in the diagram opposite.



What is the maximum arc length in centimetres (cm) that each door opening can have, so that air never flows freely between the entrance and the exit?

Maximum arc length: .... cm

Below and overleaf are two examples of items from the *Sailing Ships* unit. This unit presents mathematics problems in a scientific context. The first item, which is of medium difficulty (Level 3), falls within the Quantity mathematical content area and requires students to apply the calculation of a percentage within a real world situation. Students in Ireland performed relatively well on this item, with 60.9% of students answering it correctly, compared to an average of 59.5% across OECD countries.

Ninety-five percent of world trade is moved by sea, by roughly 50,000 tankers, bulk carriers and container ships. Most of these ships use diesel fuel.

Engineers are planning to develop wind power support for ships. Their proposal is to attach kite sails to ships and use the wind's power to help reduce diesel consumption and the fuel's impact on the environment.

One advantage of using a kite sail is that it flies at a height of 150m. There, the wind speed is approximately 25% higher than down on the deck of the ship.

At what approximate speed does the wind blow into a kite sail when a wind speed of 24km/h is measured on the deck of the ship?

A → 6km/h
B → 18km/h
C → 25km/h
D → 30km/h
E → 49km/h

The second item, below, is one of the most difficult PISA items (Level 6) and requires students to solve a real-world problem involving cost savings and fuel consumption. This is an example of an item measuring the Change & Relationships content area. Just under 16% of students in Ireland answered this question correctly, which is about the same as the OECD average of 15.3%.

Due to high diesel fuel costs of 0.42 zeds per litre, the owners of the ship ***NewWave*** are thinking about equipping their ship with a kite sail.

It is estimated that a kite sail like this has the potential to reduce the diesel consumption by about 20% overall.

Name: ***NewWave***

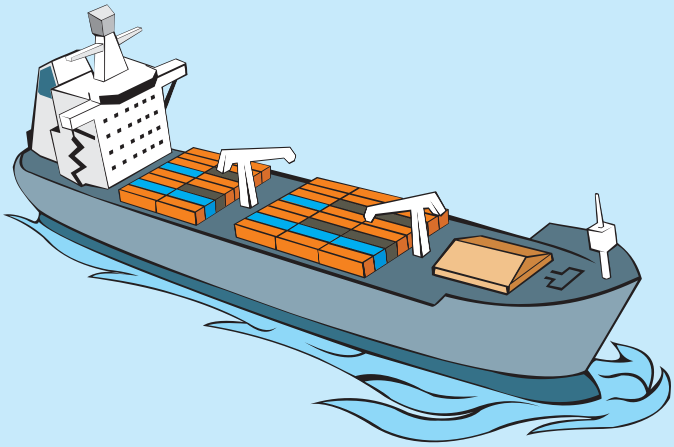
Type: freighter

Length: 117 metres

Breath: 18 metres

Load Capacity: 12,000 tons

Maximum speed: 19 knots



Diesel consumption per year without a kite sail: approximately 3,500,000 litres

The cost of equipping the ***NewWave*** with a kite sail is 2,500,000 zeds.

After about how many years would the diesel fuel savings cover the cost of the kite sail?
Give calculations to support your answer.

....

....

....

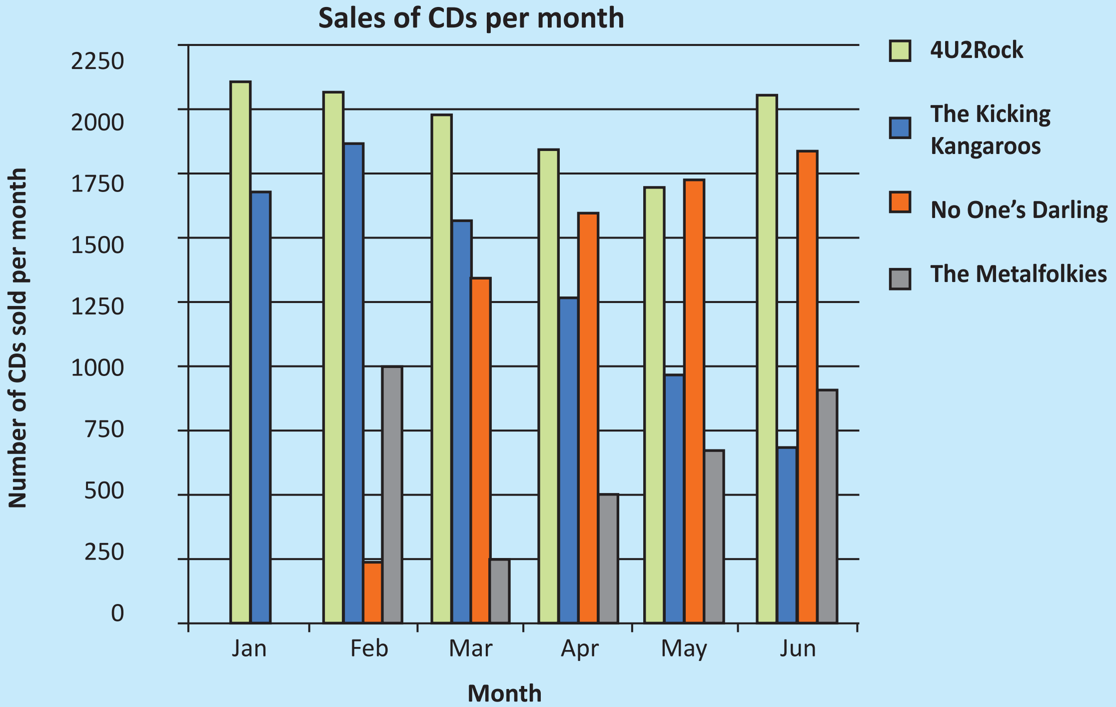
....

....

Number of years: ....

The two items from the *Charts* unit (overleaf) are examples of some of the easiest PISA items. Both items fall within the Uncertainty & Data content area and require students to read a bar chart in order to answer the questions. In the first example, students must compare the heights of two bars. This item is a PISA Level 1 item, and, although most students in Ireland answered it correctly (77%), the proportion is still lower than the average across OECD countries (80%).

In January, the new CDs of the bands ***4U2Rock*** and ***The Kicking Kangaroos*** were released.
In February, the CDs of the bands ***No One's Darling*** and ***The Metalfolkies*** followed.
The following graph shows the sales of the bands' CDs from January to June.



In which month did the band ***No One's Darling*** sell more CDs than the band ***The Kicking Kangaroos*** for the first time?

A → No month
B → March
C → April
D → May

The second example item is slightly more difficult (PISA Level 2) and requires students to interpret the bar chart and estimate the number of CDs sold in the future assuming that the linear trend continues. Seventy-six percent of students in Ireland answered this item correctly, which is about the same as the average across OECD countries (77%).

The manager ***The Kicking Kangaroos*** is worried because the number of their CDs that sold decreased from February to June.

What is the estimate of their sales volume for July if the same negative trend continues?

A → 70 CDs
B → 370 CDs
C → 670 CDs
D → 1340 CDs

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