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THE PISA MATHEMATICS RESULTS IN CONTEXT

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The performance of Irish students on the mathematics assessment of PISA 2003 is considered from the standpoints of the Irish mathematics curriculum and current issues in mathematics education. An examination of contrasting approaches to mathematics education is followed by a description of the historical roots of the Irish mathematics curriculum and some perceived current problems in mathematics achievement. This sets the context for a consideration of the overall PISA results and results on the four mathematics literacy subscales (Space & Shape, Change & Relationships, Quantity, and Uncertainty). Questions are raised regarding the direction in which elements of the Irish curriculum, in particular, approaches to teaching, learning and assessment, might evolve.

Cross-national studies of achievement provide rich data for analysis at both international and national level, but perhaps their main value for participating countries is found only when the results are examined with reference to the national context. The chief aim of this paper is to provide such a context for the Irish results from the OECD Programme for International Student Assessment (PISA) 2003 mathematics study, and to offer a detailed examination of some of the outcomes in that context. The first section of the paper describes approaches to mathematics education relevant to understanding the focus of PISA; it examines Realistic Mathematics Education and the contrasting philosophy of 'modern mathematics.' The second section identifies the approaches that have informed Irish school curricula and highlights some current issues of relevance to mathematics education. Against this background, the third section presents the main results of the PISA tests and aims to illuminate them by focusing on the performance of Irish students on individual items. In the final section, some implications for developing mathematics education in Ireland are outlined.

APPROACHES TO MATHEMATICS EDUCATION AND THEIR RELATIONSHIP TO PISA

In discussing PISA, it is important to bear in mind the nature of the study. It examines the so-called *literacy* of 15-year old students, typically at or near the end of their period of compulsory schooling, and preparing – immediately or some time later – to move to life after school. *Mathematical literacy* is defined as 'an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen' (OECD, 2003, p. 24). Such a focus naturally leads to an emphasis on applying mathematics to real-life situations and to addressing situations in which mathematics can be used to solve problems. Knowledge of mathematics as an abstract system or a cultural inheritance is, in this context, of less importance.

The philosophy underlying the approach taken to mathematics education in PISA is drawn from Realistic Mathematics Education (RME) (see, e.g., de Lange, 1996, 1998; Freudenthal, 1991; Streefland, 1991; van den Heuvel-Panhuizen, 1998; for a fuller summary in the context of PISA see Oldham, 2002). Mathematics is seen as a human activity that typically arises from real-life or other engaging contexts. Different kinds of activity are associated with different types of context and different uses of the contexts. Real-life contexts can provide either a source for the formulation of mathematics or an area of application for mathematics that has already been formulated. The process of mathematizing is important here. Horizontal mathematizing occurs when real-life problems are mapped onto appropriate mathematical structures, allowing the problem solver to move from the real world into the world of symbols. Contexts can also be used as a tool or support for the development, within the world of mathematics, of increasingly complex mathematical structures (vertical mathematizing). The essence of RME is that it involves the two components appropriately linked together. Thus, it allows for the development of (suitably grounded) abstract mathematics as well as for its application to solving real-life problems.

This dual thrust is important. The English-language name Realistic Mathematics Education is slightly misleading, and can lead to expectations that it is solely concerned with real-life or everyday applications of mathematics: payment of bills, calculation of tax returns, construction of buildings, and so forth. However, it should be noted that RME originated in the Netherlands, and is to a large extent the creation of the German mathematician Hans Freudenthal, so there are issues with regard to translation from German and Dutch thought into English. According to Dekker (2006), an RME expert from the Netherlands, the hallmark of RME is that the learner can realise what is happening; it can deal with mathematics or with fantasy as well as with practical situations, the important feature being that one can understand what is going on. Writing late in his life, Freudenthal (1991) himself said: 'I prefer to apply the term "reality" to that which at a certain stage *common sense experiences as real*' (p. 17, emphasis added). His views on implementation of the two components described above are also of interest. 'How often haven't I been disappointed by mathematicians interested in education who narrowed mathematizing to its vertical component, as well as by educationalists turning to mathematics instruction who restricted it to the horizontal one' (p. 41). In PISA, however, because of its emphasis on literacy, the horizontal component is of more relevance.

The vertical aspect of mathematizing is the hallmark of a different approach to mathematics and mathematics education: that associated with 'modern mathematics' (or 'new math') in the 1950s and 1960s (Howson, Keitel, & Kilpatrick, 1981; van der Blij, Hilding, & Weinzweig, 1980). Stemming from the work of the Bourbaki group of mathematicians in France, it considers mathematics as the study of abstract structures (sets with relations between the elements). It is presented in severely rigorous form, using precise terminology and formal logical argument. Within modern mathematics, contexts and applications are, strictly speaking, irrelevant. The approach is perhaps more suited to the re-organization and error-proofing of an existing body of knowledge than to the creative development of the subject or to early encounters with its concepts. Naturally enough, it affected many third-level mathematics programmes. In course of time, the mismatch between such programmes and curricula in schools led to pressure for 'modern' courses to be introduced at second and even at first level. Not everyone was happy with the modern mathematics movement [though, as an interesting historical footnote, Piaget (1973) was an enthusiast]. RME actually grew out of the reactions of Freudenthal and his co-workers both to it and to mechanistic or 'rote' approaches to teaching (Dekker, 2006).

The concepts of vertical and horizontal mathematizing can be used to identify four archetypal approaches to mathematics education (Treffers, 1991). Vertical mathematizing alone is associated with the structuralist curricula of modern mathematics.[1] Horizontal mathematizing alone leads to empiricist curricula; the empiricist approach is the one chiefly emphasized in PISA. Both forms of mathematizing (suitably linked) are required for curricula in the realistic tradition, as argued above. If no mathematization is involved, curricula are mechanistic, and mathematics becomes the application of rules devoid of

[1] The term structuralist can be applied to a particular development of modern curricula that involves emphasis on concept development, for example through discovery learning, rather than only on mathematical content (Howson et al., 1981). For the purposes of the categorization here, it includes the content-focused modern mathematics approach.

context or meaning. A point important for the discussion later in this paper is that the categorization of a curriculum does not depend only, or indeed chiefly, on the mathematics content (i.e., the list of topics to be taught). It has more to do with the presentation and experience of that content, and hence with teaching, learning, and assessment.

MATHEMATICS EDUCATION IN IRELAND: BACKGROUND AND CURRENT PROBLEMS

In Ireland, the second-level mathematics syllabuses revised in the 1960s were strongly affected by modern mathematics, as were many others revised at that time. Further revisions of the syllabuses in the 1970s continued the trend (Oldham, 1989). The legacy persists to the present day, contributing some topic areas which can be seen as intrinsically interesting and/or important in the development of mathematics for the 21st century, but also leaving the courses with a perhaps undue emphasis on formal notation and abstraction and insufficient emphasis on application and problem-solving in real-life contexts. A review of post-primary mathematics by the National Council for Curriculum and Assessment (NCCA) provided a long-overdue opportunity to consider the fundamental aims of mathematics education and perhaps to strike a different balance between vertical and horizontal mathematizing. The primary curriculum introduced in 1971 (Department of Education, 1971) also reflected some emphasis on mathematical structure but, being informed by Piagetian principles with regard to concept development, it did not highlight the abstract approach that typefies the modern mathematics movement. The revised primary curriculum (published in 1999 and implemented for mathematics from 2002) puts more emphasis on horizontal mathematizing, with problem solving in contexts an important thread (DES/NCCA, 1999).

Curricular intentions are not always reflected in student performance. In recent years, there has been considerable dissatisfaction with the mathematical knowledge and skills demonstrated by students at the end of post-primary schooling and on entry to third-level education. The problems were crystallized in 2001, on the one hand by the Chief Examiner's report on performance in the Ordinary-level Leaving Certificate examination (DES, 2001), and on the other hand by a report indicating that the non-completion rate in university for students taking mathematics-related courses was higher than for students in many other areas (Morgan, Flanagan, & Kellaghan, 2001). The percentage of students obtaining low scores (grade E, grade F, or no grade) in mathematics in the Ordinary-level Leaving Certificate examination in particular means that some thousands of students leave the school system each year without having achieved a grade regarded as a ‘pass’ in mathematics. Such students are in general excluded from third-level courses that require mathematical knowledge and skills. Moreover, many students who achieve ‘pass’ grades struggle with the mathematics in third-level courses; several third-level institutions now provide some form of learning support for those who need to revisit their school mathematics and to learn how to engage with, and apply, basic concepts with understanding.

This suggests that at least some of the difficulties can be traced to the culture of learning and teaching mathematics in schools in Ireland. While there are many excellent teachers and diligent students who strive to promote and achieve a sound understanding of mathematics, it can be argued that the dominant culture is not one that emphasizes mathematizing and allied skills (Oldham, 2001). Further evidence can be found by considering the ongoing tension with regard to examination design between, on the one hand, those responsible for the state examinations and, on the other hand, teachers and students. A report produced for the NCCA in 2003 commented unfavourably on the very predictable nature of the examinations, in which mathematical topics are rarely set in real-life or unusual contexts, and indeed generally appear only in familiar positions on the papers (Elwood & Carlisle, 2003). Such predictability has led to the emergence, or at least supported the existence, of newspaper articles and other summaries offering very question-specific advice for students. This advice can be parodied as ‘such-and-such a technique will be tested in Paper II, question 6, part (b) (iii)’ with the implication that if the examiners require the technique to be used in Paper II, question 6, part (b) (ii), they are guilty of a dirty trick. In other words, students are given many *non-mathematical* clues as to the techniques that may be required to answer a particular question: a situation that encourages a mechanistic approach to learning. When the state examiners introduce material in unfamiliar positions and/or innovative contexts, protests by teachers and students are featured in the media. This, in turn, creates pressure for a continuation of predictable examining and is a further disincentive to the development of a mathematizing culture.

It should be recognized that an exclusive or very dominant focus on lower-level skills is not in accordance with curricular intentions, at either Junior Certificate or Leaving Certificate level. One illustration of this can be drawn from the specifications for design of the state examinations. Questions are generally divided into three parts, labelled (a), (b), and (c); while parts (a) and (b) are expected typically to address recall, comprehension, and standard techniques, part (c) is intended to examine applications and limited problem-solving (DES/NCCA, 2002). This structure was designed to produce a balance between excessive focus on difficult problems (which does not allow students to show what they know) and excessive focus on routine (which enables students to achieve high grades without using problem-solving skills). A consequence of this design is that, once an innovative part (c) has appeared in a paper, typically such a question should not occur again, because it would no longer have the required characteristic of unfamiliarity. However, in practice, it seems that many teachers and students aiming for high grades try to cover many part (c)s as isolated examples, reducing them to rehearsed procedural tasks, rather than developing skills that would allow the students to address unfamiliar problems (Close & Oldham, 2005). This has the effect of simultaneously lengthening the course and failing to achieve the required process skills.

In a situation dominated by high-stakes examinations and the associated competition for scarce third-level places, it is easy to understand the reasons that have caused the short-term goals of mechanistic learning for state examinations to dominate over the long-term goals of meaningful learning and development of appropriate skills for life, study, and work. However, the problems identified above indicate that for many students, the short-term strategy is not succeeding. Some are 'failing' the examinations; some who 'pass' do so only via an experience that, because of its lack of meaning, alienates them from mathematics; some, who felt competent and consequently enjoyed mechanistic mathematics in school, face disillusionment and disempowerment when encountering a different style of mathematics education at third level.

The NCCA review of post-primary mathematics education was itself an outcome of the problems and led to an unprecedented amount of writing and talking about mathematics education in Ireland. The publication of a discussion paper (NCCA, 2005) preceded a public consultation, feedback from which has been summarized and presented in a report (NCCA, 2006). The NCCA also commissioned a review of international literature on curriculum and assessment, in particular for mathematics in the senior cycle, to provide information and insight with regard to approaches undertaken elsewhere (Conway & Sloane, 2005). The PISA study has been a further timely and important contribution to the discussions. Aspects of the study particularly relevant to the review are considered in the following sections.

THE IRISH RESULTS FOR MATHEMATICS IN PISA 2003

The main aim of this section is to examine how Irish students, who may well have focused their study of mathematics on doing familiar types of questions, as argued in the preceding section, performed on the very different tasks in the PISA 2003 tests. To illustrate the style of the PISA tasks and the areas that they examine, much of the focus is on individual items: in particular, on released items that have been placed in the public domain (available from *http://erc.ie/pisa/7.php*). However, the item level analysis is set in the context of a brief overview, first of key aspects used in the design and analysis of the mathematics tests, and secondly of the overall performance of Irish students on the test.

Key Concepts in PISA

The underlying philosophy of RME (Realistic Mathematics Education) in PISA is operationalized in the PISA mathematics framework (see Close, 2006). Only the aspects most relevant to the present discussion – classifications of mathematical content and process, and their relation to the Irish mathematics curriculum – are presented here. Two further concepts, those of scale score and proficiency level, are crucial to the interpretation of the results and hence are also briefly described.

The *mathematical content* examined by PISA is classified in terms of *overarching ideas* or domains (Quantity, Space & Shape, Change & Relationships, and Uncertainty), giving rise to four corresponding subscales in the tests. The overarching ideas may be considered as representing areas of mathematics as encountered in daily life and work, rather than as reflecting the traditional strands of academic mathematics or school curricula. The Test-Curriculum Rating Project indicates the degree of overlap of each domain with content areas in the Irish Junior Certificate syllabus: sets, number systems, applied arithmetic and measure, algebra, statistics, geometry, trigonometry, and functions and graphs (Cosgrove, Shiel, Sofroniou, Zastrutski, & Shortt, 2005). Predictably, most items in the Quantity domain test material from the content areas number systems and especially applied arithmetic and measure. Less obviously, perhaps, the latter is also the content area into which most items in the Space & Shape domain fall; none of the PISA items focuses on the material listed in the geometry section of the syllabus in which the emphasis is chiefly on geometry as a formal deductive system (DES/NCCA, 2000). The concepts underlying the Change & Relationships domain might seem to relate most strongly to the syllabus content areas algebra and functions and graphs, but many of the items draw primarily on material from the statistics area. Many of the items in the Uncertainty domain also relate chiefly to statistics; most of the rest deal with probability, which does not figure in the Junior Certificate syllabus. The other domains also contain items outside the Junior Certificate syllabus. Conversely, the syllabus content areas sets and trigonometry, as well as geometry, receive little or no coverage in the PISA tests. Further details, taking account of the difference between the Higher, Ordinary, and Foundation level versions of the syllabus, are provided by Cosgrove et al. (2005). Altogether, it can be seen that there is a substantial mismatch between the material examined in the PISA tests and the material in the Junior Certificate syllabus.

The *processes* deemed to be used in responding successfully to the items form three *competency clusters*: Reproduction, Connections, and Reflection. Items in the Reproduction cluster test routine procedures; items in the Connections cluster require some form of association between different content areas, situations, or methods; Reflection items typically require an element of creativity or insight. The competencies can be related to the three-part structure of questions in the state examinations, as outlined above. A part (a) is typically of straightforward Reproduction type. Part (b)s are often also of Reproduction type (albeit somewhat more complex), though some would qualify for the Connections cluster. Part (c)s should in general be of at least Connections type; the intended design did not exclude Reflection items, though the previous discussion points to the fact that few examination questions have displayed the required unpredictable and innovative characteristics.

In contrast to the overarching ideas and competencies, which are theoretical constructs, the six proficiency levels emerged from analysis of the data and are essentially a function of student performance. The following description is much simplified; a fuller account is provided by Cosgrove et al. (2005). The IRT (item response theory) scaling used in PISA maps the performance of students and of items on to the same scale. The mean is set to 500 and the standard deviation to 100. Thus, for example, a student with a scale score of 550 has achieved a result half a standard deviation above the mean – in qualitative terms a good but not outstanding result. An item with a scale score of 550 is a moderately difficult item, in general answered correctly only by students with a scale score of around 550 or higher. This score is in the range classified as being at level 4, and a student who performed at this level has shown reasonable competence and some problem-solving ability.

Overview of the Irish Mathematics Results

The mean mathematics score of Irish students in PISA 2003 was 502.8, which is not significantly different from the OECD mean of 500 (Cosgrove et al., 2005; Shiel, Sofroniou, & Cosgrove, 2006). This echoes the situation in PISA 2000, when the Irish mathematics mean of 502.9 was likewise not significantly different from the OECD mean (Shiel, Cosgrove, Sofroniou, & Kelly, 2001). The verdict on these performances can perhaps be rendered in report-card form as 'could do better'. Countries with mean scores significantly higher than that of Ireland include those in the Pacific Rim (traditionally high scorers in cross-national studies of achievement in mathematics) (Beaton, Mullis, Martin, Gonzales, Smith, & Kelly, 1996; Lapointe, Mead, & Askew, 1992; Martin, Hickey, & Murchan, 1992), the Netherlands (the home of RME, and hence burdened with the expectation of successful performance), and Finland (overall, taking account of outcomes in reading and science as well as mathematics, the most successful country in PISA 2003). Countries scoring at the same level as Ireland include France and Germany. Countries scoring significantly lower than Ireland include Hungary [interestingly, a rather successful performer in the earlier and differently-focused cross-national study, TIMSS (Beaton et al., 1996)] and the USA.

The mean scores suffice to rank order the countries, but do not reveal important differences in the distribution of students' scores. Ireland's standard deviation for mathematics in 2003, at 85.3, was one of the lowest. Again this is consonant with the findings from PISA 2000, in which the Irish standard deviation was 83.6. Examination of the proficiency levels for the 2003 results gives another view of this 'bunching' of Irish scores. The percentage of Irish students whose scale scores were at levels 5 or 6 was lower than the OECD country average percentage; that is, there were relatively few high-fliers in the Irish sample. However, there were also relatively few with scale scores at or below level 1, indicating that the lower scorers from Ireland did manage to display more knowledge than the lower scorers in many other countries.

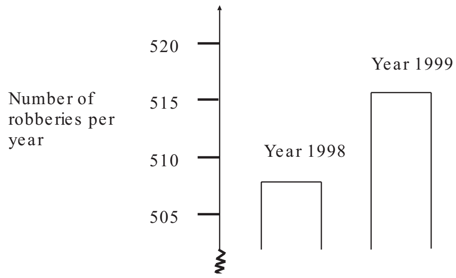
Since the mean and dispersion of the Irish scores in PISA 2003 essentially reinforce the findings from 2000, it is perhaps of most interest to focus on results that were only available in 2003: performance on the four subscales corresponding to the domains of Quantity, Space & Shape, Change & Relationships, and Uncertainty.[2] In each case, the overall result for the subscale is presented; performances on one or two units – scenarios providing contexts and associated questions – are then examined in relation to the Irish and other curricula and cultures. The subscales are addressed in descending order of Irish mean performance.

The Uncertainty Subscale

Ireland's mean score of 517.2 on this subscale is significantly above the OECD mean of 502.0. Items contributing to this score include ones from the units called 'Robberies' and 'Earthquake'; they are described in turn.

[2] In 2000, only two domains were tested, and subscales and proficiency levels were not developed until the 2003 data were available.

Example 1: Robberies
A TV reporter showed this graph to the viewers and said:
“The graph shows that there is a huge increase in the number of robberies from 1998 to 1999.”



Do you consider the reporter’s statement to be a reasonable interpretation of the graph? Give an explanation to support your answer.

Key: *Full credit*: “No, not reasonable”. Focuses on the fact that only a small part of the graph is shown; *partial credit*: “No, not reasonable”, but explanation lacks detail, or “No, not reasonable”, with correct method but with minor computational errors; *no credit:* No, with no, insufficient or incorrect explanation, yes, other responses, missing.
Process: *Connections*. Focus on an increase given by an exact number of robberies in absolute and relative terms; argumentation based on interpretation of data.

PISA Item Difficulty
Scale score: 576.7 (PC); 694.3 (FC)
Level: 4 (PC); 6 (FC)

Item statistics	% OECD	% Ireland
Fully correct	15.4	13.3
Partially correct	28.1	36.7
Incorrect	41.5	38.1
Missing	15.0	11.9
Total	100	100

The one item in this unit was difficult for students, especially with regard to gaining full credit (proficiency level 4 for partial credit and 6 for full credit). The percentage of Irish students gaining at least partial credit is somewhat greater than the OECD country average. This may reflect the fact that, on the one hand, the material is on the syllabus, but that, on the other hand, the interpretation of misleading graphs has not generally been emphasized in textbooks or examinations. Students due to sit for their Junior Certificate examination in 2003 (a few months after taking the PISA tests) or later may have been prepared for giving verbal explanations for their answers, as this is a feature of the revised course examined for the first time in 2003; students who sat for the examinations before 2003 would probably have been unaccustomed to giving explanations.

Example 2: Earthquake
A documentary was broadcast about earthquakes and how often earthquakes occur. It included a discussion about the predictability of earthquakes.
A geologist stated: "In the next twenty years, the chance that an earthquake will occur in Zed City is two out of three."

Which of the following best reflects the meaning of the geologist's statement?
A $^2/_3$ x 20 = 13.3, so between 13 and 14 years from now there will be an earthquake in Zed City.
B $^2/_3$ is more than $^1/_2$, so you can be sure there will be an earthquake in Zed City at some time during the next 20 years.
C The likelihood that there will be an earthquake in Zed City at some time during the next 20 years is higher than the likelihood of no earthquake.
D You cannot tell what will happen, because nobody can be sure when an earthquake will occur.

Key: *Full credit:* C; *no credit*: other responses, missing.
Process: *Reflection*

PISA Item Difficulty
Scale score: 557.2
Level: 4

Item statistics	% OECD	% Ireland
Fully correct	46.5	51.4
Incorrect	44.2	41.2
Missing	9.3	7.4
Total	100	100

This unit tests probability, which is not on the Junior Certificate course (and was not on the primary school curriculum at the time at which the students were in primary school). Moreover, the item is classified as being in the Reflection cluster, and it was argued above that items of Reflection type are in general unfamiliar to Irish students. Despite these facts, Irish students did rather better than OECD students overall.

This raises a more general issue. The above-average score on the Uncertainty subscale was obtained despite the fact that many items are outside the syllabus. Hence, some of the concepts and procedures examined were unlikely to have been taught to students. However, it should be noted that probability is outside the syllabus for students in some of the other participating countries also

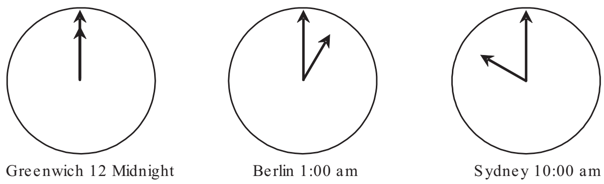
(Oldham, 2002). Moreover, the language of probability is – probably! – part of the Irish culture to a much greater extent than is the case in many other countries; so, far from operating under difficulties, Irish students may well have been advantaged over many of their international colleagues in this area.

The Change & Relationships Subscale

The Irish mean score of 506.0 on this scale is significantly above the OECD mean of 498.8. Items contributing to the scale include ones from units on ‘Internet Relay Chat’ and ‘Walking’.

Example 3: Internet Relay Chat

Mark (from Sydney, Australia) and Hans (from Berlin, Germany) often communicate with each other using “chat” on the Internet. They have to log on to the Internet at the same time to be able to chat. To find a suitable time to chat, Mark looked up a chart of world times and found the following:



Question 1

At 7:00 pm in Sydney, what time is it in Berlin?

Answer: ....

Key: *Full credit:* 10 am or 10:00; *no credit*: other responses, missing.
Process: *Connections*.

PISA Item Difficulty
Scale score: 533.1
Level: 3

Item statistics	% OECD	% Ireland
Correct	53.7	50.1
Incorrect	42.7	48.1
Missing	3.5	1.8
Total	100	100

Question 2
Mark and Hans are not able to chat between 9:00 am and 4:30 pm their local time, as they have to go to school. Also, from 11:00 pm till 7:00 am their local time they won't be able to chat because they will be sleeping. When would be a good time for Mark and Hans to chat? Write the local times in the table.

Place	Time
Sydney	
Berlin	

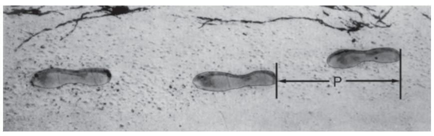
Key: *Full credit:* any time or interval of time satisfying the 9 hours time difference and taken from one of these intervals [details supplied]; no credit: other responses, including one time correct but corresponding time incorrect, missing.
Process: *Reflection.*

PISA Item Difficulty
Scale score: 635.9
Level: 5

Item statistics	% OECD	% Ireland
Correct	28.8	37.2
Incorrect	52.1	53.5
Missing	19.2	9.3
Total	100	100

The response patterns for question 2 in particular are of interest. Irish students performed rather strongly in comparison with the cohort as a whole, and were much less inclined to omit the item. This occurred despite the fact that the problem posed in the question is not common in Irish textbooks or examinations, so students were unlikely to know a routine procedure that would yield a correct answer. One can speculate that, compared with students in some countries, they were familiar with making contacts across time zones, perhaps to relatives in the USA or Australia – with mobile phones if not by means of internet chat – and may have been particularly inclined to engage with the problem.

Example 4: Walking



The picture shows the footprints of a man walking. The pace length P is the distance between the rear of two consecutive footprints. For men, the formula, $n/P = 140$, gives an approximate relationship between n and P where n = number of steps per minute and P = pace length in metres.

If the formula applies to Mark's walking and Mark takes 70 steps per minute, what is Mark's pace length? Show your work.

Key: *Full credit*: 0.5 m or 50 cm, ½ (unit not required); *partial credit*: 70/ p = 140, 70 = 140 p, p = 0.5. 70/140; *no credit:* other responses, missing.
Process: *Reproduction.* Reflect on and realise the embedded mathematics, solve the problem successfully through substitution in a simple formula, and carry out a routine procedure.

PISA Item Difficulty
Scale score: 611.0
Level: 5

Item statistics	% OECD	% Ireland
Fully correct	36.3	22.9
Partially correct	21.8	34.7
Incorrect	20.9	28.1
Missing	21.0	14.3
Total	100	100

Example 4 is an item that has been classified as of Reproduction type but was found to be difficult; hence, it may illustrate the fact that the relationship between item type and item difficulty is not simple. However, for Irish students the item is not routine. While it tests material on at least the Junior Certificate Higher-level syllabus, the occurrence of the unknown in the denominator removes it from the realm of often-rehearsed procedures. The proportion of students obtaining full credit is predictably low, but in terms of obtaining at least partial credit, the Irish performance is in line with that of the OECD cohort. The data again illustrate the tendency for Irish students to be more ready than average at least to supply an answer, even if incorrect.

The Quantity Subscale

The Irish mean score of 501.7 on the Quantity subscale does not differ significantly from the OECD mean of 500.7. Items contributing to the scale include ones from the unit entitled 'Skateboard'.

Example 5: Skateboard
Eric is a great skateboard fan. He visits a shop called SKATERS to check some prices. At this shop you can buy a complete board. Or you can buy a deck, a set of 4 wheels, a set of 2 trucks and a set of hardware, and assemble your own board. The prices for the shop's products are:

Product	Price in zeds	
Complete skateboard	82 or 84	
Deck	40, 60 or 65	
One set of 4 Wheels	14 or 36	
One set of 2 Trucks	16	 
One set of hardware (bearings, rubber pads, bolts and nuts)	10 or 20	  

Question 1
Eric wants to assemble his own skateboard. What is the minimum price and the maximum price in this shop for self-assembled skateboards?

(a) Minimum price: _______________ zeds.
(b) Maximum price: _______________ zeds.

Key: *Full credit:* both the minimum (80) and the maximum (137) are correct; *partial credit:* only the minimum (80) is correct, or only the maximum (137) is correct; *no credit:* other responses, missing. Process: *Reproduction.* Find a simple strategy to come up with the maximum and minimum, use of a routine addition procedure, use of a simple table.

PISA Item Difficulty
Scale score: 463.7 (PC); 496.5 (FC)
Level: 2 (PC); 3 (FC)

Item statistics	% OECD	% Ireland
Fully correct	66.7	69.0
Partially correct	10.6	8.2
Incorrect	18.0	20.8
Missing	4.7	2.0
Total	100	100

Question 2
The shop offers three different decks, two different sets of wheels and two different sets of hardware. There is only one choice for a set of trucks.
How many different skateboards can Eric construct?

A 6
B 8
C 10
D 12

Key: *Full credit:* D; *no credit:* other responses, missing.
Process: *Reproduction.* Interpret a text in combination with a table correctly; apply a simple enumeration algorithm accurately.

PISA Item Difficulty
Scale score: 569.7
Level: 4

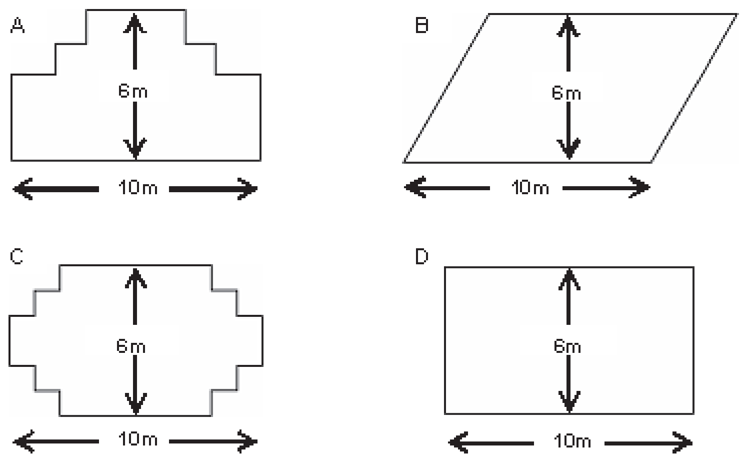
Item statistics	% OECD	% Ireland
Correct	45.5	30.2
Incorrect	50.0	66.9
Missing	4.5	2.9
Total	100	100

This unit can be considered as presenting archetypal PISA tasks. The introductory scenario involves pictures; moreover, knowledge of the context may well be helpful, though not actually necessary, in addressing the problem. The first question, of Reproduction type, was fairly easy for Irish students, as it was for OECD students in general. However, Irish students did poorly on question 2. This is not surprising because the enumeration algorithm required is on the Leaving Certificate rather than the Junior Certificate course, and so would have been unknown to most of the group.

The Space & Shape Subscale

The Irish mean score on this subscale was 476.2, significantly below the OECD mean of 496.3. Two units, ‘Carpenter’ and ‘Number Cubes,’ are examined to illustrate this less than average performance.

Example 6: Carpenter



A carpenter has 32 metres of timber and wants to make a border around a vegetable patch. He is considering the following designs for the vegetable patch.

Circle either "Yes" or "No" for each design to indicate whether the vegetable patch can be made with 32 metres of timber.

Vegetable patch design	Using this design, can the vegetable patch be made with 32 metres of timber?
Design A	Yes / No
Design B	Yes / No
Design C	Yes / No
Design D	Yes / No

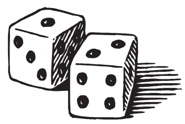
Key: *Full credit*: four correct (yes, no, yes, yes, in that order); *partial credit*: three correct; *no credit:* two or fewer correct, missing.
Process: *Connections.* Use geometrical insight and argumentation skills, and possibly some technical geometrical knowledge.

PISA Item Difficulty
Scale score: 687.3
Level: 6

Item statistics	% OECD	% Ireland
Fully correct	20.0	13.0
Partially correct	30.8	30.9
Incorrect	46.8	54.6
Missing	2.5	1.6
Total	100	100

This was a difficult item for PISA students in general, and particularly for Irish students. Interestingly, it is a rare example of an item for which the formal study of traditional Euclidean geometry (technical geometrical knowledge), more emphasized in the Irish curriculum than in some others, might have proved helpful, particularly in identifying the fact that the slant sides of the non-rectangular parallelogram are greater than 6m in length; but few students made the required connections. Skills of visualization might have proved equally helpful, but these are not greatly featured in the Irish curriculum.

Example 7: Number Cubes



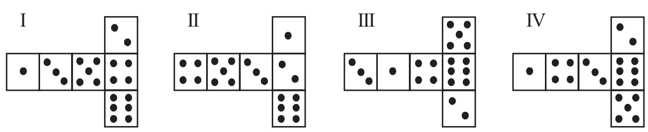
On the right, there is a picture of two dice.

Dice are special number cubes for which the following rule applies:

The total number of dots on two opposite faces is always seven. You can make a simple number cube by cutting, folding and gluing cardboard. This can be done in many ways.

In the figure below you can see four cuttings that can be used to make cubes, with dots on the sides.

Which of the following shapes can be folded together to form a cube that obeys the rule that the sum of opposite faces is 7? For each shape, circle either “Yes” or “No” in the table below.



Shape	Obeys the rule that the sum of opposite faces is 7?
I	Yes / No
II	Yes / No
III	Yes / No
IV	Yes / No

Key: *Full credit:* No, yes, yes, and no, in that order; *no credit:* other responses, missing.
Process: *Connections.* Encode and interpret 2-dimensional objects, interpret the connected 3-dimensional object, and check certain basic computational relations.

PISA Item Difficulty
Scale score: 503.5
Level: 3

Item statistics	% OECD	% Ireland
Correct	63.0	57.4
Incorrect	34.7	40.9
Missing	2.3	1.7
Total	100	100

This item requires knowledge of the net of a cube (not on the Irish curriculum) or use of visualization skills (not emphasized in Ireland). The below-average performance on a moderately easy item is thus consistent with expectations based on the Irish curriculum.

The Irish results from PISA 2003 in this area are similar to the relatively poor Irish performances on geometry or space/shape elements of previous cross-national studies (see, e.g., Lapointe, Mead, & Phillips, 1989; Martin et al., 1992). In general, in these studies, there has been a tendency for the type of geometry that featured in the Irish curricula at the time to be under-represented and for the types that did not to be over-represented in the tests.

CONCLUSION

Two basic questions are addressed in conclusion. What can we learn from PISA? And to what extent should an RME approach be adopted in post-primary mathematics education in Ireland?

One short answer to the first question stems from a consideration of the PISA framework and test items, without any reference to the performance of Irish students on the tests. It is that mathematics education can be different – different in style from that which appears to be the Irish second-level norm, as described earlier. In particular, examinations can be different from current state examinations. This fact is illustrated by the items presented in this paper, items which place more emphasis on the solution of problems embedded in engaging contexts than on the execution of technical procedures. While PISA itself does not explicitly address teaching and learning styles, its approach to assessment would seem to necessitate a change in these respects also: from predominantly mechanistic practice (on the part of at least some, and perhaps many, Irish teachers and students) to one that encourages mathematization.

A second answer reflects Irish performance. In the report-card language used earlier, by comparison with their peers in other countries, Irish students in general, and higher-achieving Irish students in particular, 'could do better,' while lower-achieving Irish students (who out-performed their lower-achieving peers in many countries) 'could do worse.' The relatively poor performance of the top Irish students is a matter for concern. It appears that they have not been given the tools to address unfamiliar problems well; perhaps they have even been conditioned not to address such problems, and have acquired a 'learned helplessness' in this regard.

With regard to lower-achieving students, the situation looks more encouraging. Further analysis would be needed to establish exactly the source of their scores and where these exceeded performance by low achievers in other countries. In the meantime, several hypotheses may be put forward. One is based on the assumption that, because of the technical and formal emphasis in the Irish curriculum, the lower-achieving Irish students have experienced more mathematical content and techniques than is the case for many lower-achieving students elsewhere (a conjecture that would have to be confirmed by an up-to-date curriculum analysis for all participating countries); if so, perhaps they have benefited from at least some of their experiences. A second hypothesis is that the test items were so unfamiliar to the students that they had few preconceived barriers to the possibility of success. A third hypothesis refers to the amount of reading required to address the items, and suggests that the achievement reflected in the relatively strong performance of Irish students in the reading component of PISA may have advantaged students in the mathematics component (Cosgrove et al., 2005). The comparatively small number of students in the Irish cohort for whom the tests were not in their first language could be a contributory factor here. A fourth hypothesis, with some support from data analysis, relates to the comparatively large number of Irish students absent on the day of the tests; students likely to perform poorly may have been over-represented among absentees (Cosgrove, 2005).

Confirmation of either or both of the last two hypotheses would have negative implications for the interpretation of Ireland's overall performance. In that case, perhaps the report-card entry could be adjusted to state: 'could do better, especially in light of participating students' ability to read the test questions.' Counterbalancing factors, however, are the unfamiliar style of the tests and the rather small amount of time given to mathematics in the Irish school curriculum (Oldham, 2002). Another version might, therefore, be 'Irish students have done quite well in view of the constraints imposed by the school and subject curriculum and by the teaching-learning culture.'

This leads to consideration of priorities for the curriculum and hence to addressing the second question above: should RME receive greater emphasis in the Irish curriculum? In addressing the question, the intention is not so much to provide a specific answer as to draw attention to important and relevant issues. In the past, Department of Education syllabus committees and their successors, the NCCA course committees, have tended to focus chiefly on mathematical content – on whether given topics should be included or excluded – and on comparatively minor changes to the structure and format of the state examinations. They gave less consideration to classroom culture and to teaching and learning styles, and in the last 40 years did not critique the underlying paradigm; such issues were, or may have been deemed to be, outside their brief (Oldham & English, 2005). Similar comments can be made about the discussions, for example at mathematics teachers' meetings, which helped to inform the work of such committees. There is some evidence that that situation has been changing (Oldham & English, 2005). Recent discussions in the public domain (in responses to the NCCA consultation process, at meetings of mathematics teachers and other professional groups, and in newspaper articles) have been wider ranging and deeper than heretofore.

This scope and depth can be illustrated by examining the report describing feedback from the consultation process (NCCA, 2006). While the design of the questionnaire to structure the process doubtless affected the outcome, and while the responses (about 300) are not necessarily representative, the report gives the best available indication of current thinking on the part of teachers, lecturers, students, and other interested bodies and individuals. Many thoughtful individual and group submissions are quoted. Not surprisingly, calls were made for change in curriculum content; however, there does not appear to have been consensus on the details. If specific mathematical topics were commonly targeted as candidates for inclusion or exclusion, they are not highlighted in the document. Rather, the emphasis was on broad issues such as greater focus on applicability and less on abstraction. Critiques of teaching, learning, and especially assessment provide a major theme in respondents' answers: for example, looking for emphasis on understanding, real-life applications, modelling, or problem-solving approaches. Nonetheless, not all responses favoured a curricular revolution. With regard to content, there was a measure of satisfaction with the (recently revised) Junior Certificate syllabus. Some respondents emphasized positive features of abstract mathematics, for which, so to speak, the motivation, context, and applications are provided by mathematics itself. The case was also advanced for some rote learning. Moreover, respondents referred to difficulties for teachers in adopting a radically different approach to their teaching, as would be required if the style of the curriculum were to be much changed; substantial ongoing professional development would be needed. Some contributions pointed to external constraints, such as the short time available for mathematics in an overloaded school curriculum. In general, however, the report indicates that many respondents are willing at least to consider, and perhaps to endorse, quite fundamental change.

The issues raised can be set in broader context. Naturally, syllabus content should be critiqued at intervals with regard to its current relevance. In this respect, it is noteworthy that in recent years the author has received several personal communications to the effect that the existing content is fairly satisfactory and that curriculum problems lie elsewhere. This echoes the point, made earlier, that content is not the chief determinant of curriculum style. However, if an RME-type approach is to be introduced successfully without extra time being allocated to mathematics, the content will have to be reduced to allow the relevant process skills to be addressed. With regard to rote learning, a happier description might be that there is still a need for fluent performance, built on understanding of concepts and appropriate practice (Cockcroft, 1982). For some teachers and students, focusing on understanding would be a change from current practice; for many, adopting a problem-solving approach in the tradition of RME would be a major shift. Radical change in the state examinations, incorporating questions like those presented in this paper, might force some such changes, but the difficulty of implementing them successfully should not be underestimated.

This point can be considered further. With regard to teachers, a recent study by Kaiser (2006) highlights such problems for teachers whose beliefs about mathematics are not consonant with RME. Using the fourfold classification of curricula as introduced above (different from, but showing clear relationships with, the classification of teacher beliefs in Kaiser's paper), it can be seen that inappropriate teaching can reduce approaches intended to be realistic to ones that are mechanistic (devoid of mathematization), just as happened all too often with structuralist approaches. Supposedly realistic curricula can also degenerate into empiricist ones, in which the emphasis is on horizontal mathematizing, with consequent adverse effects on advanced mathematics. There is some indication that this has happened in The Netherlands, heretofore generally regarded as having implemented the realistic approach successfully. The emergence of difficulties was confirmed by Hans Pelgrum, a Dutch veteran of mathematical and other cross-national studies, in a personal communication to the author (1 April 2006).

The fact that successful change might be difficult does not mean that it should not be attempted. If the goals are felt to be sufficiently important, then appropriate resources should be deployed to support the initiative. Reports from a project which introduced RME in a small number of schools in England indicate that the well-disposed and well-supported teachers in the project are enjoying the approach, and that most, though not all, participating students prefer it.[3] The success of the implementation, in particular with regard to vertical mathematizing, cannot be judged for some time to come, but a promising start has been made. It would be good if Irish students could benefit from the enthusiasm and engagement generated by such a project without losing the advantages of encountering more technical mathematics. Perhaps this would help higher-achieving students to face unfamiliar challenges and, where relevant, to progress, well equipped, to third-level education in mathematical and scientific areas. It might also allow lower achievers to apply the technical skills they actually possess in contexts that they feel they can understand. However, much work would need to be done to design and implement a curriculum that would seek to provide the best of both worlds.

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[3] The conference 'Developing Maths in Context: Making Sense of Mathematics,' held in Manchester, on July 6-7, 2006, included reports on, and discussion of, the project, notably by participating teachers and lecturers.

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