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THE PERFORMANCE OF IRISH STUDENTS IN MATHEMATICAL LITERACY IN THE PROGRAMME FOR INTERNATIONAL STUDENT ASSESSMENT (PISA)

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Ways in which the results of PISA 2000 for mathematics can contribute to our understanding of the strengths and weaknesses of mathematics education in Ireland are considered. Explanations of Ireland's performance are sought in terms of the study's distinctive scope and style. Since these reflect 'realistic mathematics education' (RME), a philosophy of mathematics education rather different from that prevailing in Ireland, the origin and features of RME are examined in some detail. Attention is then focused on PISA 2003. The evolution of the framework for the mathematics element of the study is described, and areas of consistency and of change with respect to the framework for PISA 2000 are identified. Finally, implications for mathematics education in Ireland are considered.

The Programme for International Student Assessment (PISA) comes at an important time for Ireland with regard to mathematics education. A revised primary-school curriculum, with increased emphasis on problem-solving, was introduced in 2002. The introduction of a revised Junior Certificate Mathematics syllabus (examined for the first time in 2003) was supported by an incareer development programme that sought to diversify approaches to teaching and learning in mathematics classrooms. A review of the Leaving Certificate syllabus is currently under way. Altogether, issues to do with teaching and learning, rather than just curriculum content, are being considered to a greater extent than heretofore.

Mathematics was a minor domain in PISA 2000 and a major domain in 2003. It is now possible to look back at the 2000 results and to formulate questions about mathematics education in Ireland in the light of the findings. This paper is concerned not so much with the fine details of Irish performance as with the broad issues raised by the study. It aims to consider ways in which they can help us to understand the strengths and weaknesses of mathematics education in Ireland. In the first section of the paper, PISA 2000 test results and explanations of the Irish performance in terms of the study's distinctive scope and style are considered. The philosophy of mathematics education underlying the study is sufficiently different from the prevailing philosophy in Ireland to merit extended discussion, and this is provided in the second section. In the third section, attention is focused on PISA 2003. Finally, implications for mathematics education in Ireland are considered.

PISA 2000 MATHEMATICS

In this section, the main achievement results on the mathematics component of PISA 2000 are described. The performance of Irish students is then set in context by a consideration of the distinctive rationale, content, and style of the PISA tests.

Irish Performance in PISA 2000

The mean score for Irish students did not differ significantly from the overall mean of participating countries. Of the 27 OECD countries that met the specifications for sampling and data collection, Ireland's mean score was ranked fifteenth. This compares unfavourably with the results for reading (in which Ireland scored significantly above the overall country mean and was ranked 5th), and to a lesser extent for science (in which Ireland's ranking of 9th again corresponded to a mean score that was significantly above the overall country mean). Moreover, the comparatively weak performance in just one area was unusual; in general, countries that performed strongly in one area performed strongly in all three (Shiel, Cosgrove, Sofroniou, & Kelly, 2001). Thus, the results in mathematics may appear somewhat disappointing.

PISA: Rationale, Content, and Style

The rationale, content, and style of PISA are all important in considering the performance of Irish students. As regards *rationale*, PISA – unlike many cross-national studies [such as the Third International Mathematics and Science Study (TIMSS) (Beaton, Mullis, Martin, Gonzales, Smith, & Kelly, 1996)] – did not aim to reflect national curricula. Rather, it tested what were believed to be the knowledge and skills needed for life after school. The 'mathematical literacy' examined by PISA is defined as

> an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded mathematical judgements and to engage in mathematics, in ways that meet the needs of that individual's current and future life as a constructive, concerned and reflective citizen. (OECD, 1999, p. 41)

In line with this rationale, mathematical *content* for PISA 2000 was considered, not so much in terms of traditional topic areas (algebra, geometry, and so forth), as in terms of *themes* that might be encountered or experienced in everyday life: shape and space, change and growth, quantitative reasoning, chance, uncertainty, and dependency and relationships (OECD, 1999). Since mathematics was a minor domain in 2000, only the first two of these themes were tested. 'Shape and space' – obviously encompassing aspects of geometry – sounds relatively familiar to those involved in Irish mathematics education at junior-cycle level. However, 'change and growth' may not, suggesting a focus on calculus or on other areas of mathematics beyond the scope of junior-cycle students. In fact it is intended to refer, inter alia, to patterns, relationships, functions, graphs, and statistics. All of these occur in the junior-cycle curriculum (Department of Education and Science/National Council for Curriculum Assessment, 2000), but in practice the emphasis is not so much on applications involving ideas of change as on formal and technical skills.

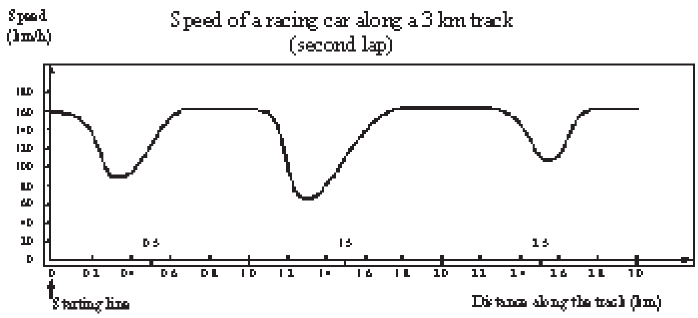
Formal and technical skills are recognized in PISA, but the emphasis is on higher order thinking. A set of eight skills or *competencies* was given for PISA 2000: mathematical thinking; mathematical augmentation; modelling; problem-posing and problem-solving; representation; symbolic, formal, and technical skill; communication; and use of aids and tools (OECD, 1999). For working purposes, such as the construction of test items, a more useful construct is that of what in PISA 2000 were called *competency classes*: class 1, carrying out standard routine procedures; class 2, making connections and integrating ideas for the purposes of problem-solving; and class 3, involving mathematical thinking, generalization, and insight (OECD, 1999). Comparison with questions on Irish examination papers is of interest here. For the Leaving Certificate and for the revised Junior Certificate examination papers, questions in general are expected to have a 'gradient of difficulty' that is related to the assessment objectives of the syllabus. Questions typically fall into three sections, an easy 'part (a),' a routine or standard 'part (b)' and a more challenging problem or application for 'part (c)' (Department of Education and Science/National Council for Curriculum and Assessment, 2002, pp. 92, 96-100). Most of the 'part (a)' and 'part (b)' sections would be placed in PISA's class 1, while the 'part (c)' might be in Class 2.

Considering the *rationale* and *content* of PISA, and the emphasis on higher order objectives, it is not surprising that the general *style* of questions in the PISA tests differs markedly from that in Irish textbooks and examination papers. Contrasting examples are given by way of illustration. Figure 1 shows one of the clusters of items released from the PISA 2000 tests. It starts with a short scenario, referring to motor racing – a topic which may catch the interest of typical 15-year old students – and displaying a graph of speed against distance.

Figure 1
Example of PISA Questions

Unit: Speed of Racing Car
(Context: Personal; Big Idea: Growth and Change; Maths Strand: Functions)

This graph shows how the speed of a racing car varies along a flat (level) 3 kilometre track during its second lap.



Question 4
Aspect:Class 2; multiple-choice
PISA Scale Score = 665.
Here are pictures of five tracks:
Along which one of these tracks was the car driven to produce the speed graph shown earlier?

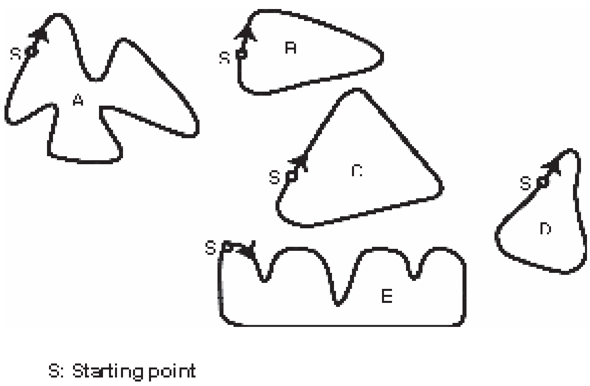


Figure 1 (cont.)

Percent Choosing Each Response	*Ireland*	*OECD*
A	20.9	16.9
B*	18.5	28.6
C	3.8	7.7
D	5.1	5.3
E	50.7	37.5
Missing	1.0	4.0

*Key

Question 5
Aspect: Class 1; multiple-choice
PISA Scale Score = 413.
Where was the lowest speed recorded during the second lap?
A at the starting line.
B at about 0.8 km.
C* at about 1.3 km.
D halfway around the track

Percent Choosing Each Response	*Ireland*	*OECD*
A	7.2	5.6
B	2.4	2.8
C*	83.5	83.4
D	4.5	6.3
Missing	2.4	1.9

*Key

Question 6
Aspect: Class 1; multiple-choice
PISA Scale Score = 423.
What can you say about the speed of the car between the 2.6 km and 2.8 km marks?
A The speed of the car remains constant.
B* The speed of the car is increasing.
C The speed of the car is decreasing.
D The speed of the car cannot be determined from the graph.

Percent Choosing Each Response	*Ireland*	*OECD*
A	4.8	4.4
B*	81.4	82.7
C	8.2	7.4
D	4.7	3.9
Missing	0.9	1.7

*Key

Figure 1 (Contd.)
Question 7
Aspect: Class 2; multiple-choice
PISA Scale Score = 502.
What is the approximate distance from the starting line to the beginning of the longest straight section of the track?
A 0.5 km
B* 1.5 km
C 2.3 km
D 2.6 km

Percent Choosing Each Response	*Ireland*	*OECD*
A	10.9	11.5
B*	66.0	66.9
C	11.6	10.7
D	9.9	7.7
Missing	1.6	3.2

*Key

Source: Shiel et al, 2001, pp. 187-188.

Students are perhaps expected to have some background knowledge, not entirely mathematical, which they can apply to the question: racing cars slow down when going round corners. Students are asked a series of questions, all related to the scenario but independent of each other. Item statistics from PISA 2000 show that the first question, in particular, turned out to be challenging, and Irish students did poorly on it. The percentage of Irish respondents who confused the shape of the graph with the shape of the race-track was considerably higher than the OECD mean percentage. Interpreting graphs in this way is not a skill emphasized in the Irish syllabus. This is a class 2 question. Irish students' performances on the other three questions (two of which were judged to be in class 1 and one of which was again considered as falling into class 2) were very close to the OECD averages.

The items in Figure 2 are parts of questions in the Junior Certificate examinations for 2000. They are decontextualized, and the algebra questions can be regarded as demanding considerable abstract skill relative to the assumed ability level of the target groups. Some Junior Certificate questions do require students to consider information in a context, but the 'story line' is usually minimal and students would not need extra-mathematical knowledge such as that required in the motor racing question.

Figure 2
Examples of Questions from the Junior Certificate Examinations, 2000

A 'part (a)' from the Higher level paper 1

Using the same axes and scales, draw the graphs of
$f: x \rightarrow 3 + x - 2x^2$
$g: x \rightarrow 1 - x$
in the domain $-2 \leq x \leq 3, x \in \mathbf{R}$.

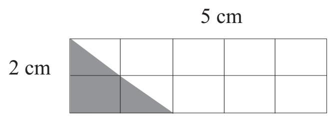
A 'part (b)' from the Higher level paper 1

Solve for *x*:

$$\frac{1}{(x-5)(x+3)} - \frac{3}{x-5} = \frac{2}{x+3}, \quad x \neq 5, \;\; x \neq -3.$$

A typical question from the Foundation level paper

(i) What is the area of the diagram on the right?
(ii) What is the area of the shaded part?
(iii) What fraction of the diagram is shaded?



An algebra question from the Foundation level paper

(i) Find the value of x for which $2x - 5 = 7$.
(ii) Find the value of $3x + 7y$ when $x = 2$ and $y = 1$.

That PISA-style questions are unfamiliar to Irish students is corroborated by a small-scale Irish study in which the mathematics items from PISA 2000 were examined by a team familiar with the style of items in Irish textbooks and examination papers (Shiel et al., 2001). Moreover, Irish students – those following the Higher level syllabus, in particular – would have spent time studying a considerable amount of material, even in the areas designated by the two themes that figured in PISA 2000, that was not examined in PISA (Department of Education and Science/National Council for Curriculum and Assessment, 2000). In view of this, Ireland's performance can perhaps be seen in a rather different light. An intrinsically very strong performance (with high percentages of correct responses on many items) would have been surprising.

Whether or not a performance that was strong relative to other participating countries might have been expected is a rather different question. It depends, inter alia, on whether the other countries were likely to be equally unfamiliar with the PISA approach and to have devoted similar proportions of their time to topics not assessed in PISA.

The PISA mathematics tests reflect a philosophy of mathematics education which is currently very prominent in the international literature and is known as 'realistic mathematics education' (de Lange, 1994; de Lange, 1998; Streefland, 1991; van den Heuvel-Panhuizen, 1998). Related philosophies or dominant views about mathematics education emphasize problem-solving (National Council of Teachers of Mathematics, 2000; Schoenfeld, 1992) and so-called 'authentic' assessment: assessment that (for instance) provides meaningful tasks rather than artificial exercises, and genuinely reflects the skills associated with the mathematics being tested (Mueller, 2003; National Council of Teachers of Mathematics, 1995). It has to be said that, at least until recently, unlike the situation in other countries, debate about mathematics education in Ireland has been largely untouched by these issues (Oldham, 2001). An understanding of realistic mathematics education and its associated ideas is important in interpreting PISA results, and will be considered below.

So far, discussion has focused on mean scores; no account has been taken of dispersion. However, it might be conjectured that the best students would rise to the challenge posed by the unfamiliar style and would obtain high scores, but that weaker students would be unable to start the questions and would perform poorly. This proved to be wrong. Irish students scoring at the national 10th percentile (that is, students below whose scores 10% of Irish scores lay) achieved well above the OECD country mean score for the 10th percentile (Shiel et al., 2001, Table 3.8). However, Irish students scoring at the national 90th percentile did relatively poorly, being ranked 20th among 27 participanting countries at this level.

In Ireland, the mean score of males was significantly higher than that of females, and 'the data suggest that more males than females are represented at the higher end [of the distribution]' (Shiel et al., 2001, Table 4.1, Table 4.4). In view of recent concerns about male underachievement, these trends – although not strong – are worthy of closer examination. The non-standard and non-routine style of the PISA questions may be a contributing factor.

The PISA policy with regard to calculators was that students who normally used them should be allowed to do so. Some quarter of the Irish students had access to a calculator during the tests, and these performed significantly better than students without access (Shiel et al., 2001, p. 86). PISA questions were intended to be calculator-neutral (in the sense that use of a calculator was not expected to be of benefit). Perhaps the relatively good performance of the Irish calculator-access students was due to some other variable or variables; if not, then the calculator-neutral standing of the PISA items is called into question. The issue is discussed further below.

REALISTIC MATHEMATICS EDUCATION (RME)

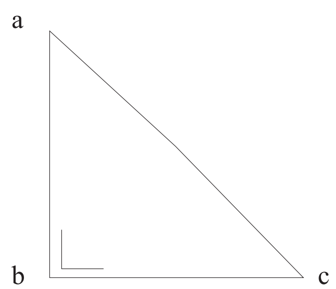
In this section, 'realistic mathematics education' (RME) (which underpins the mathematical component of PISA) is discussed. Some of its characteristics have been highlighted by the examples given above. Another group of examples is provided to emphasize its main features, which are then outlined. Finally, the position of the Irish curriculum with regard to realistic mathematics education is examined.

Examples of Realistic and Non-Realistic Mathematics Questions

Two questions on the topic of *measure* are presented in Figure 3. One shows a right-angled triangle abc with sides of given length; the other shows an identical figure labelled cps. In the first question, the student is asked to find the area of the triangle. The second question is effectively the same, but is wrapped up in a brief story line about a village in which there are a church, a post office (or perhaps, with higher probability, a public house), and a school; the student has to find the area of the triangle with the three buildings as vertices. The first example deals with abstract mathematics. The second provides a *context*, but gives no reason for wanting to find the area of the triangle. This is not a 'realistic' question; the context acts merely as a 'decoration' (Oldham, van der Valk, Broekman, & Berenson, 1999; van der Valk & Broekman, 1997), perhaps serving only to delay engagement with the mathematical essentials. Assuming that the students are familiar with the mathematics required, both examples are routine; in neither case is there a challenging problem to be examined and solved.

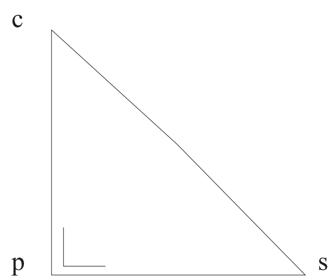
A different kind of example might be formulated round a scenario in which the annual school play is being produced. The set designer wants a triangular region (of given dimensions) of the stage to be marked out as a lawn, as illustrated in Figure 4. The local shop is selling shaggy green matting; the students have to work out how much to buy to make the lawn. In this case, the answer is not necessarily the area of the proposed lawn. The students have to consider the form in which the matting is sold (e.g., in strips of a given width), the possibility of making the triangle out of a number of pieces of matting to avoid wastage, and so forth. Their eventual conclusion is to be accompanied by reasons, and there could be more than one acceptable answer. A question of this kind may perhaps be considered 'realistic.'

Figure 3
Examples of Questions Dealing with the Area of a Triangle



In the triangle abc, |ab| is 500 units and |bc| is 400 units; $\angle$ abc is a right angle

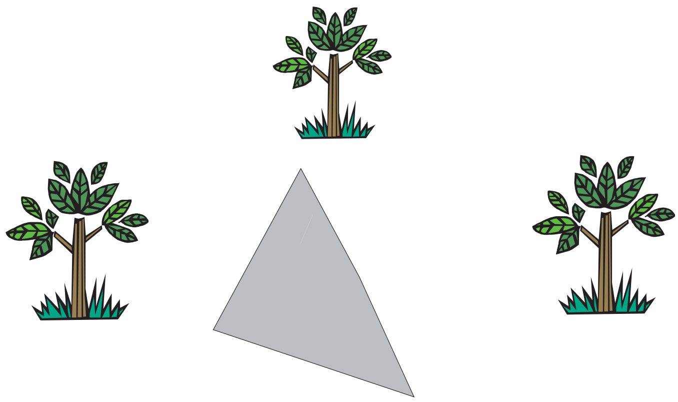
Find the area of triangle abc.



In the village of Ballybeg are a church (c), a post office(p) and a school (s). The church is 500 yards North of the post office; the school is 400 yards East of the post office (see diagram

Find the area of triangle cps

Figure 4
Basic Diagram for a Question on Buying Matting for the School Play



The Characteristics of Realistic Mathematics Education

Realistic mathematics education arose as a reaction to the 'modern mathematics' movement of the 1960s (van den Heuvel-Panhuizen, 1998). 'Modern mathematics' (or 'new math') treated the subject as the study of *structures*; it emphasized precise definitions, rigorous arguments, and abstraction (Howson, Keitel, & Kilpatrick, 1982, pp. 100-107; van der Blij, Hilding, & Weinzweig, 1980). Even as a philosophy of mathematics, rather than of mathematics education, it was controversial. Its supporters valued its clarity and rigour and the way in which it reconstructed mathematics logically as an integrated whole. However, people who regarded mathematics as a problem-solving activity drawing on intuition (rather than as a set of structures), or as a tool for understanding the real world, were not likely to be favourably disposed to the modern mathematics movement. In any case, there were difficulties when the 'modern mathematics' structuralist approach was transferred from university to school level. It required what in Piagetian terms would be regarded as formal operational thinking, a form of thinking not necessarily developed by students at the ages at which they were introduced to the work. In the words of the German mathematics educator Hans Freudenthal (1983), '[the] wrong perspective of the so-called new math was that of replacing the learner's by the adult mathematician's insight' (p. 4).

Freudenthal was the person chiefly responsible for developing RME. He worked in the Netherlands, and influenced Dutch mathematics education from around the 1970s; through the seventies and eighties he was the mastermind at what is now, in his honour, called the Freudenthal Institute in Utrecht (Streefland, 1991). In 1980, he was a plenary speaker at the Fourth International Congress on Mathematical Education, held in Berkeley, California. His lecture discussed 13 problems in mathematics education, including: Why can [a particular child] not do arithmetic? How do people learn? How can one keep open the sources of insight? How can suitable contexts be created in order to teach 'mathematizing'? (Freudenthal, 1983). These questions point to a number of crucial features of RME.

Of primary importance is the fact that in RME mathematics is regarded as a human activity (Freudenthal, 1968). It is consonant with a constructivist approach to learning, and typically involves going from a concrete situation to abstract concepts which are then suitably applied (de Lange, 1994). Thus, it uses *contexts* as sources for learning mathematics. The choice of appropriate concepts is crucial (de Lange, 1998). In many cases, they seem to be found empirically: by trial and error, or trial and improvement, in actual classrooms. Obviously, this can lead to difficulties: a context of interest to one person may be of no interest to another, and a context relevant in one culture may make no sense in another. Perhaps the race-track example, described above, would not have been suitable if PISA had included countries with very low car ownership.

The term 'mathematizing' is often applied to the modelling-type procedure in which a real-life problem is translated into a mathematical form in which it can be solved and the solution is appropriately interpreted. Two kinds of mathematizing are involved in RME. 'Horizontal mathematizing' occurs when examples are found in the real world with similar mathematical structure to that of the concept under consideration, allowing the learner to move from the real world into the world of symbols; in a classroom setting, it might be measured by the extent to which use is made of contexts and concrete materials. 'Vertical mathematizing' is concerned with developing mathematical structures, and might be measured by the number of aspects of a mathematical concept that are addressed (Oldham, et al., 1999; Treffers, 1991a; van den Heuvel-Panhuizen, 1998). Structuralists (those espousing 'modern mathematics') focus on vertical mathematizing. It is relevant here to mention a further group: mechanists – those who do not engage in any kind of mathematizing, but see mathematics education in terms only of rules to be learned ('invert and multiply,' 'add a 0,' 'take it over and change the sign,' and so forth). Unfortunately, structuralist curricula implemented by teachers who do not have a clear awareness of their underlying rationale can become mechanist in practice; this is seen as one reason why 'modern mathematics' curricula did not fulfil their early promise in schools (Oldham et al., 1999: van der Blij et al., 1980).

RME may make undue demands on teachers (see Treffers, 1991b). Other problems can be identified. Even de Lange (1994), who inherited Freudenthal's role as the mastermind in the area, has raised some possible difficulties: loss of basic skills and routines, loss of structure, and loss of the kind of clarity with which examination-focused systems are very familiar, together with the complexity of implementing the genuinely 'authentic' assessment that matches the aims of RME. However, he claims that the gradual and co-ordinated introduction of RME in the Netherlands has been attended by success, as shown, for example, by Dutch performance in TIMSS (Beaton et al., 1996; de Lange, 1998).

RME and Irish Mathematics Education

The philosophy underlying the revised primary-school mathematics curriculum (Department of Education and Science/National Council for Curriculum and Assessment, 1999b), with its explicit focus on problem-solving and on use of contexts, is quite close to that of RME. (The degree of closeness and the extent to which the Irish curriculum arrived at the philosophy without formal contact with the literature on RME are matters of interest.) At second level, Ireland adopted 'modern mathematics' in the 1960s to a greater extent than many other countries (Oldham, 1989; Oldham, 2001); the syllabuses and, in particular, the examination papers, as indicated by the examples in Figure 2, still reflect the focus on precise terminology and abstraction that is characteristic of the movement. The recent revision of the Junior Certificate syllabus was less profound than the revision at primary-school level; it was essentially a minor update to deal with areas of the course that were giving rise to difficulties, rather than a root-and-branch review. However, the incareer development programme accompanying introduction of the revised syllabus represents an attempt to move away from mechanist approaches towards teaching for understanding (Department of Education and Science/National Council for Curriculum and Assessment, 2002). Similar issues are likely to arise in the forthcoming review at senior-cycle level.

PISA 2003

Some important aspects of the form of the mathematics component of PISA 2003 were discussed at a PISA forum attended by the author. However, final decisions were taken elsewhere and are encapsulated in the current version of the 'framework' (OECD, 2003)[1]. An account of the forum will be followed by a description of key aspects of the framework.

The Mathematics Forum

Ireland was not the only country to experience a poor match with the prevailing philosophy of PISA. Some of the issues were discussed at a specially convened Mathematics Forum. Each participating country can send one mathematics educator to the Forum, which is also attended by key members of the PISA consortium and, where possible, by members of the Mathematics Expert Group that provides leadership, insight, and expertise in mathematics education to the study. The author of this paper has had the privilege of representing Ireland at the Forum.[2] It met three times in the period 2000-2001. The first meeting was chiefly concerned with discussing aspects of the rationale

1 Since the framework document had not been published at the time of writing this paper, references to it are based on an advanced draft. There may be minor discrepancies between the two versions.

2 One member of the Mathematics Expert Group, Dr Seán Close, is from Ireland, but is there in his own right and not as an Irish representative.

and framework of the study; the second two were devoted chiefly, though not exclusively, to considering sets of items that might be used in assessment tasks.

Four major issues were discussed at the first Forum meeting (held in December 2000): the underlying RME philosophy that dominates PISA, and three less fundamental issues – the age group addressed in the study, the relationship with national curricula, and the policy with regard to use of technology.

The Forum endorsed PISA's adoption of an RME approach. In the opinion of the author, there were three main reasons. First, the approach is intrinsically appropriate to a consideration of the 'mathematical literacy' that PISA aims to assess. Secondly, it is topical and of considerable intrinsic interest in the world of mathematics education, as indicated above. Thirdly, there would be little point in conducting a more traditional type of study when that had recently been done – and redone, and is to be done yet again – by TIMSS. However, it was recommended that some concessions would need to be made to countries for which the RME approach was alien. For example, some easy questions with minimal contexts might be included to allow students to get started on the assessment tests.

There could be little profitable discussion about the age group involved in the study since the matter was a fait accompli. However, given that many of the participating countries retain many of their students in school well past the age of 15, the use of the 15-year olds to measure the output of school systems with regard to general (mathematical and other) literacy can be regarded as inappropriate. In a system with a high retention rate, for example, it is possible that curricula in the junior cycle of second-level education would emphasize academic knowledge, and that the task of preparing students to apply that knowledge in everyday circumstances would be addressed at senior cycle. Given that no change in the age of assessment is possible, it will be a matter for individual poorly-scoring countries to consider whether or not the goals espoused by PISA are, or should be, covered in the higher grades of their education systems.

As indicated earlier, PISA did not focus on testing common curricular material (identified, for example, in a cross-national curriculum analysis). Rather, the tests aimed to assess mathematical knowledge and skills needed in life after school. At the first meeting of the Forum, some time was spent trying to reassure members that PISA was not sitting in judgment on their national curricula (T. Romberg, address at the Forum meeting, December 2000). However, two issues arose: Who determines what knowledge and skills are required for mathematical literacy as defined by PISA? And if such knowledge and skills are *not* included in national mathematics curricula, and if they are genuinely relevant for mathematical literacy, why are they omitted? The first point highlights the fact that PISA tests are not value-free; they represent some people's judgments of what mathematics is needed for 'constructive, concerned and reflective' citizenship as described in the definition of mathematical literacy (OECD, 1999, p. 41). Hence, the choice of content and skills assessed in PISA is open to critique. The second point also serves to highlight the need for critique, this time by individual countries to account for any gaps in their curricula. It should be noted that countries may well be able to justify any such gaps in the light of their own educational priorities. Cross-national studies are not intended to be normative, though unfortunately they are sometimes treated as if they are (Bishop, 1993). However, seeing one's own curriculum in some kind of international context can be very fruitful in alerting one to different possibilities. Whether these possibilities should give rise to change at national level is a matter for reasoned debate at that level.

The remaining issue of interest arising from the initial Forum meeting and discussed here was that of the use of aids and tools for the tests, and in particular the role of technology. It was noted earlier that the PISA guideline for the tests in 2000 was that students could use calculators if they were accustomed to doing so (and hence might be disadvantaged by their absence). However, in countries – or, more locally, in schools – in which policy is opposed to calculator use, students would have to manage without. This is not consistent with the 'realistic' philosophy of PISA. Admittedly, items were meant to be calculator-neutral, so that the presence or absence of calculators should not be of great relevance. However, the Irish results throw some doubt on the genuine calculator-neutral standing of the items. Moreover, calculator-neutral questions are scarcely 'authentic'; in real life, awkward numbers are the norm. The idea was raised at the Forum that, with the role of technology in the world growing, PISA might take a lead in requiring the provision of calculators (not necessarily in requiring the students to use them if the students preferred not to do so), and might even look at more innovative forms of technology-based assessment of mathematics such as the use of spreadsheets in solving problems.

It transpired that the use of technology was not such a fundamental point in PISA's interpretation of mathematical literacy as was the use of contexts. In the current version of the framework, it is again stated that students should use calculators as they are normally used in school; otherwise the students accustomed to calculator use would be disadvantaged. This effectively allows students to be denied access to calculators if that is the local policy. The parallel argument that students unused to solving problems in context will be disadvantaged by the extensive use of context-embedded problems in the tests was not given the same credence. An intermediate position, which is discussed below, may be adopted with regard to content areas.

The PISA Framework for 2003

The mathematics framework for PISA 2003 is built round the general concept of mathematizing, as described above. In the proposed framework (OECD, 2003), it is recognized that authentic assessment of mathematizing for the solution of real-life problems should involve group work taking place over an extended period of time; the restriction to timed pen-and-paper tests is basically inappropriate. However, for a data collection exercise on the scale of PISA, it is probably inevitable. The process of mathematization is split into components which can be considered to some extent separately. This is helped by the way that the framework distinguishes situations and contexts; content; and competencies and competency clusters. Problems occur in some broad *situations* of relevance to students' lives. They are at different 'distances' from the students. Nearest to the students are situations of concern to their personal lives; then comes school life; then work and leisure; then the local community and society; and finally scientific situations. Thus, PISA problems are not restricted to 'everyday' or 'practical' applications such as those that figure in the 'applied arithmetic and measure' sections of the Junior Certificate syllabus (Department of Education and Science/National Council for Curriculum and Assessment, 2000). They can even include some so-called intra-mathematical problems (where the relevant situations are considered as a subset of the scientific category), provided that these are likely to have the appropriate appeal for the student cohort (van den Heuvel-Panhuizen, 1998). The emphasis, however, is on extra-mathematical situations. Not only is a particular problem set in a situation, it has a specific *context* which typically is identified by the 'story line' introducing the problem. Thus, in the 'matting lawn' question (Figure 4), the situation might be identified as school drama – hence part of school life – and the context as being concerned with buying appropriate properties.

The *content* 'themes' are now named 'overarching ideas'. They have been reduced in number to four: quantity, space and shape, change and relationships, and uncertainty. From this classification, certain points arise for discussion. 'Space and shape' – as for the equivalent category in PISA 2000 – obviously encompasses geometric ideas. However, as is usual for the geometric component of international studies (see, for example, Lapointe, Mead, & Phillips, 1989, Figure 3.7), the match with the Irish junior-cycle curriculum is poor; in particular, it does not cover the formal, deductive approach, emphasizing theorems and proofs, that dominates the Higher level of the Junior Certificate syllabus (Department of Education and Science/National Council for Curriculum and Assessment, 2000). This serves to illustrate a debate which arose at the first Forum meeting as to whether the mathematics in PISA was broader or narrower than that in school curricula. The lack of emphasis on formal geometry and on the use of technology are restrictions compared with some national curricula; however, the range of situations and contexts used tends to be wider.

A further issue, prefigured in the discussion of calculators and contexts above, is raised by 'uncertainty.' One important aspect of uncertainty is probability, a topic that in several participating countries, including Ireland, is not addressed at junior-cycle level (Department of Education, 1992; Department of Education and Science/National Council for Curriculum and Assessment, 2000). This serves to illustrate a debate which arose at the first Forum meeting 2000). However, it is one of the most obviously applicable topics to real-life situations. The extent to which such a topic should appear in the tests was discussed at later Forum meetings (held in 2001). Forum members seemed to be in favour of a compromise whereby it would appear, but with less weighting than might be merited by its real-life importance. Another point of interest is that probability is a topic on which students sometimes do better than their teachers expect; they can attempt questions on the basis of general knowledge even when they have not addressed the topic in school.

A consideration of *competencies* and *competency clusters* completes the description of key aspects of the framework. The clusters (not 'classes,' as in PISA 2000) are named as 'reproduction,' 'connections' and 'reflection,' avoiding the hierarchical numbering system used in the 2000 study. The clusters are not intended to form a hierarchy with regard to difficulty. There can be difficult reproduction items and easy reflection items. However, the framework document acknowledges that, in general, there is likely to be a gradient of difficulty across the three clusters (OECD, 2003).

IMPLICATIONS FOR MATHEMATICS EDUCATION IN IRELAND

It is clear that the mathematics domain of PISA 2003 is designed to have the same basic characteristics as that of PISA 2000. It reflects the philosophy of realistic mathematics education, in which mathematics is regarded as a human activity rather than as the study of abstract structures, and in which problems are embedded in suitable contexts that are intended to engage the interest of the learners as well as allowing them to develop their understanding of mathematics.

Questions, therefore, are likely to be rich in text and informal in tone, differing from those familiar to Irish students. However, most of the mathematical content is likely to figure in the Irish Junior Certificate syllabus, though some topics, such as probability, may provide exceptions.

Most Irish students in the 15-year old group targeted by PISA are in the junior cycle. With the implementation of the revised Junior Certificate syllabus, these students should be a little better prepared for the PISA tests in 2003 than was the case in 2000. The revised syllabus shares with PISA an emphasis on students' ability to communicate their findings. The incareer development programme accompanying introduction of the syllabus emphasized meaningful learning rather than a mechanistic focus on 'rules.' Moreover, since the revised syllabus explicitly includes calculator use, all Irish students in the junior cycle should now have access to calculators and experience of using them appropriately. The results from PISA 2000 suggest that this will confer some advantage. In practice, however, PISA 2003 probably comes a little too soon to reflect benefits of the revised curriculum.

Nonetheless, PISA 2003 should help us to monitor our progress. This progress may or may not be reflected by a very different position in the ultimate rank-ordering; other countries may also be benefiting from reforms and becoming more used to 'realistic' approaches. Our satisfaction, or otherwise, with the outcomes should depend among other things on the extent to which we choose to prioritize the goals endorsed by PISA vis-à-vis those reflected in studies such as TIMSS.

It should also take into account more general educational priorities. Among the issues that arise is that of the time allocated to mathematics (both in absolute terms and as a proportion of the overall time spent in school). OECD (2002) statistics report 'intended instruction time' for the 9-11 and 12-14 age-groups, as a percentage of total instruction time, in Ireland as 12% for the former group and about 12.5% for the latter (pp. 284-285). However, as time allocations are made at school rather than at national level, and total instruction time varies across countries, these figures are of limited value in comparing the actual time typically allocated to mathematics in Irish schools to that typically allocated elsewhere. Some school-level data regarding the situation in Ireland are available from the late 1990s. The study of the junior cycle by the National Council for Curriculum and Assessment (1999) reported that 47.8% of responding schools assigned less than what might have been considered the standard five periods per week to mathematics in first year, and 18.3% assigned less than five periods (in all cases four periods) per week in third year (p. 93). When the junior certificate syllabus was revised, the Higher level content in particular was reduced to allow for more meaningful learning in the time available (Department of Education and Science/National Council for Curriculum and Assessment, 2002). However, anecdotal evidence suggests that some schools have reacted by decreasing their allocation of periods. At primary level, Shiel and Kelly (2001, p. 102) estimated that, in fourth class in 1999, the mean time given to mathematics per week was 4 hours and 10 minutes. Since guidelines for implementing the revised primary-school curriculum suggest a minimum allocation of three hours per week for mathematics (Department of Education and Science/National Council for Curriculum and Assessment, 1999a, p. 70), the time devoted to mathematics in primary school may also fall in the years ahead. If time for mathematics is reduced, then the task of covering the present content while emphasizing the communication, meaningful learning, and problem-solving characteristic of 'realistic' approaches will be more difficult.

Ultimately, the chief value of large-scale cross-national studies does not come from the publication of international reports and 'league tables' in which total scores are rank-ordered, though these may serve as catalysts for useful discussion. The true worth of such studies emerges when the results are examined in local context, drawing on detailed knowledge of educational priorities, national curricula, and classroom cultures. It emerges also from follow-up studies in which data are examined in more detail than is possible for a summary international report, and the opportunity is taken to reflect on the meaning of the results. In particular, it is manifested when groups informed by the findings from the studies engage at national or local level in meaningful debate about the goals, strengths, and weaknesses of their own education systems.

REFERENCES

Beaton, A. E., Mullis, I. V. S., Martin, M. O., Gonzales, E. J., Smith, T. A., & Kelly, D. L. (1996). *Mathematics achievement in the middle school years: IEA's Third International Mathematics and Science Study*. Chestnut Hill, MA: Center for the Study of Testing, Evaluation, and Educational Policy, Boston College.

Bishop, A. J. (1993). Influences from society. In A. J. Bishop, K. Hart, S. Lerman, & T. Nunes (Eds.), *Significant influences on children's learning of mathematics* (pp. 3-26). Paris: UNESCO.

de Lange, J. (1994). Curriculum change: An American-Dutch perspective. In D.F. Robitaille, D. H. Wheeler, & C. Kieran, (Eds.), *Selected lectures from* *the 7th International Congress on Mathematical Education* (pp. 229-248). Sante-Foye, Canada: Les Presses de L'Université Laval.

de Lange, J. (1998). Real problems with real world mathematics. In C. Alcina, J. Alvarez, M. Niss, A. Pérez, L. Rico, & A. Sfard (Eds.), *Proceedings of the 8th International Congress on Mathematical Education* (pp. 83-110). Seville: S.A.E.M THALES.

Department of Education. (1992). *The Leaving Certificate: Mathematics syllabus*. Dublin: Stationery Office.

Department of Education and Science/National Council for Curriculum and Assessment. (1999a). *Primary school curriculum: Introduction*. Dublin: Stationery Office.

Department of Education and Science/National Council for Curriculum and Assessment. (1999b). *Primary school curriculum: Mathematics*. Dublin: Stationery Office.

Department of Education and Science/National Council for Curriculum and Assessment. (2000). *Junior Certificate Mathematics syllabus (Higher, Ordinary and Foundation level).* Dublin: Stationery Office.

Department of Education and Science/National Council for Curriculum and Assessment. (2002). *Mathematics Junior Certificate: Guidelines for teachers.* Dublin: Stationery Office.

Freudenthal, H. (1968). Why to teach mathematics so as to be useful. *Educational Studies in Mathematics*, *1*, 3-8.

Freudenthal, H. (1983). Major problems of mathematics education. In M. Zweng, T. Green, J. Kilpatrick, H. Pollak, & M. Suydam (Eds.), *Proceedings of the Fourth International Congress on Mathematical Education* (pp. 1-7). Boston: Birkhäuser.

Howson, G., Keitel, C., & Kilpatrick, J. (1982). *Curriculum development in mathematics.* Cambridge: Cambridge University Press.

Lapointe, A. E., Mead, N. A., & Phillips, G. W. (1989). *A world of differences: An international assessment of mathematics and science*. Princeton NJ: Educational Testing Service.

Mueller, J. (2003). *Authentic assessment toolbox*. *http://jonathan.mueller.faculty.noctrl.edu/ toolbox/whatisit.htm*

National Council of Teachers of Mathematics. (1995). *Assessment standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

National Council for Curriculum and Assessment. (1999). *The junior cycle review—Progress report: Issues and options for develoment.* Dublin: Author.

OECD (Organisation for Economic Co-operation and Development). (1999). *Measuring student knowledge and skills: A new framework for assessment.* Paris: Author.

OECD. (2002). *Education at a glance: OECD indicators.* Paris: Author.

OECD. (2003). *The PISA 2003 assessment framework. Mathematics, reading, science and problem solving knowledge and skills.* Paris: Author.

Oldham, E. (1989). Is there an international mathematics curriculum? In B. Greer & G. Mulhern (Eds.), *New directions in mathematics education* (pp. 185-224). London: Routledge.

Oldham, E. (2001). The culture of mathematics education in the Republic of Ireland: Keeping the faith? *Irish Educational Studies*, *20*, 266-277.

Oldham, E., van der Valk, T., Broekman, H., & Berenson, S. (1999). Beginning pre-service teachers' approaches to teaching the area concept: Identifying tendencies towards realistic, structuralist, mechanist or empiricist mathematics education. *European Journal of Teacher Education, 22,* 23-43.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition and sense making in mathematics. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334-370). New York: Macmillan.

Shiel, G., Cosgrove, J., Sofroniou, N., & Kelly, A. (2001). *Ready for life? The literacy achievements of Irish 15-year-olds with comparative international data*. Dublin: Educational Research Centre.

Shiel, G., & Kelly, D. (2001). *The 1999 National Assessment of Mathematics Achievement*. Dublin: Educational Research Centre.

Streefland, L. (Ed.) (1991). *Realistic mathematics education in primary school.* Utrecht: Freudenthal Institute.

Treffers, A. (1991a). Didactical background of a mathematics program for primary education. In L. Streefland (Ed.), *Realistic mathematics education in the primary school* (pp. 21-56). Utrecht: Freudenthal Institute.

Treffers, A. (1991b). Realistic mathematics education in the Netherlands 1980-1990. In L. Streefland (Ed.), *Realistic mathematics education in the primary school* (pp. 11-20). Utrecht: Freudenthal Institute.

van den Heuvel-Panhuizen, M. (1998). *Realistic mathematics education*. Based on a lecture given at Kristiansand, Norway, 5-9 June 1998. *http://www.fi.uu.nl/en/rme/ welcome.html.*

van der Blij, F., Hilding, S., & Weinzweig, A. I. (1980). A synthesis of national reports on changes in curricula. In H.-G. Steiner (Ed.), *Comparative studies of mathematics curricula: Change and stability 1960-1980*. Proceedings of a conference jointly organized by the Institute for the Didactics of Mathematics (IDM) and the International Association for the Evaluation of Educational Achievement (IEA). *Materialen und Studien Band 19* (pp. 37-54.1). Bielefeld: Institüt für Didaktik der Mathematik der Üniversität Bielefeld.

van der Valk, T., & Broekman, H. (1997). Dutch pre-service teachers' ideas on teaching mathematics. In J. McCall & R. Mackay (Eds.), *Partnership and cooperation*. Papers from the 21st Annual Conference of the Association for Teacher Education in Europe (ATEE) held at the University of Strathclyde, Glasgow, September 1996 (pp. 103-120). University of Strathclyde, Faculty of Education, Glasgow.