



PISA Mathematics: A Teacher's Guide

Gerry Shiel

Rachel Perkins

Seán Close

Elizabeth Oldham



AN ROINN DEPARTMENT OF
OIDEACHAIS EDUCATION
AGUS EOLAÍOCHTA AND SCIENCE



PISA Mathematics: A Teacher's Guide

Gerry Shiel
Rachel Perkins
Seán Close
Elizabeth Oldham

Prepared for the Department of Education and Science by the Educational Research Centre



Copyright © 2007, Department of Education and Science

Cataloguing-in-Publication Data

Shiel, Gerry

PISA mathematics: a teacher's guide/

Gerry Shiel, Rachel Perkins, Seán Close and Elizabeth Oldham.

vi, 60p; 30 cm

Includes bibliographical references.

1. Programme for International Student Assessment (Project)
2. Mathematics (Post-primary) – Ireland
3. Educational Surveys – Ireland
4. Syllabus

2007

I Title. II Perkins, Rachel. III Close, Seán. IV Oldham, Elizabeth.

Designed by Boyd Freeman Design (www.boydfreeman.ie)

Published by the Stationery Office, Dublin

To be purchased directly from the
Government Publications Sales Office,
Sun Alliance House,
Molesworth Street, Dublin 2

or by mail order from
Government Publications,
Postal Trade Section,
51 St. Stephen's Green, Dublin 2
Tel: 01-647 6834
Fax: 01-647 6843

€8

ISBN: 0-7557-7586-4

Table of Contents

Preface	v
1 What is PISA?	1
2 How does PISA assess mathematics?	4
3 What is a PISA mathematics question like?	7
4 How did students in Ireland perform on PISA mathematics?	19
5 How does PISA mathematics compare with Junior Certificate mathematics?	25
6 How do student and school characteristics relate to performance on PISA mathematics?	31
7 Do teachers in Ireland teach PISA mathematics?	39
8 What can we learn from PISA mathematics?	45
References	49
Glossary of Terms	51

Preface

The Programme for International Student Assessment (PISA) is an international assessment of the skills and knowledge of 15-year olds. A project of member countries of the Organisation for Economic Co-operation and Development (OECD), it takes place at three-year intervals. In 2003, the main focus in PISA was mathematics, while reading, science and cross-curricular problem solving were emphasised to a lesser extent. The focus of this guide is on the performance of 15-year olds in Ireland in mathematics in 2003, relative to their counterparts in other participating countries. The guide also examines factors associated with students' performance in mathematics, and consider similarities and differences between PISA mathematics and Junior Certificate mathematics. This guide is an adaptation of the main PISA 2003 report for Ireland and is intended for teachers of Junior Certificate students in post-primary schools in Ireland.

In Ireland, PISA is jointly implemented by the Department of Education and Science and the Educational Research Centre. In March 2003, 3,880 students in 141 Irish post-primary schools took part. Similar numbers of students participated in the assessment in 40 other countries. The students completed tests of mathematics, reading, science and cross-curricular problem solving, and completed a questionnaire. Their principal teachers also completed a questionnaire. In Ireland, but not in other participating countries, a questionnaire was also completed by the students' mathematics teachers.

This guide is divided into 8 chapters. The first provides an overview of PISA, and establishes a context by detailing recent initiatives in mathematics education in Ireland. The second looks at how PISA assesses mathematics. The third provides examples of the types of items that appeared in the PISA mathematics assessment. The fourth details the performance of students in Ireland in PISA mathematics. The fifth compares the PISA mathematics framework with the Junior Certificate mathematics syllabus, and examines the performance of Irish students in PISA who sat the Junior Certificate mathematics examination in either 2002 or 2003. Chapter six looks at school and student characteristics associated with PISA mathematics. Chapter seven details the outcomes of the questionnaire administered to mathematics teachers of students in PISA 2003. Chapter eight reflects on the outcomes of PISA 2003 mathematics, and provides suggestions for applying the PISA approach to teachers and learning mathematics.

Readers who would like more detailed information on PISA 2003 mathematics than is provided here are referred to OECD (2003, 2004) and Irish (Cosgrove et al., 2005) reports on the survey and to relevant journal articles (e.g., Close, 2006; Oldham, 2006).

Acknowledgements

The help of the PISA National Advisory Committee in developing this guide is gratefully acknowledged. In addition to the authors of the guide, the committee members were Doreen McMorris (Department of Education and Science, Chair), Declan Kennedy and Tom Mullins (University College Cork), Alison Graham (Sandford Park School, Dublin), Bill Lynch (National Council for Curriculum and Assessment), and Nick Sofroniou, Eemer Eivers and Carly Cheevers (Educational Research Centre). The comments of Thomas Kellaghan, Director of the Educational Research Centre, and of inspectors of the Department of Education and Science, on earlier drafts of this guide are also gratefully acknowledged.

1 What is PISA?

PISA is an international assessment of the skills and knowledge of 15-year olds. It is a project of the Organisation for Economic Co-operation and Development (OECD) and takes place at three-year intervals. In the first assessment, which took place in 2000, reading literacy was the **major assessment domain**¹, and mathematics and science literacies were **minor domains**. In 2003, mathematics literacy was the major domain, while reading literacy, science literacy, and cross-curricular problem solving were minor domains. This guide presents the results of the PISA 2003 mathematics assessment and considers implications for teaching and learning mathematics in post-primary schools in Ireland. We begin by considering recent developments in mathematics education in Ireland.

Recent developments in mathematics education in Ireland

A revised Junior Certificate mathematics syllabus (DES/NCCA, 2000) was implemented in schools in 2000 and examined for the first time in 2003. Therefore, the results of PISA 2003 provide a valuable opportunity to consider new developments in Junior Certificate mathematics in an international context. The revised syllabus has two aims which are common to all three syllabus levels: to contribute to the personal development of students; and to help to provide them with the mathematical knowledge, skills and understanding needed for continuing their education, and eventually for life and work. The syllabus features some key changes from its predecessor, including the use of calculators in mathematics classes (and, by extension, in the Junior Certificate mathematics examination). The related document, *Junior Certificate Mathematics: Guidelines for Teachers* (DES/NCCA 2002), suggests that changes in content be accompanied by an increased emphasis on developing relational understanding, on the communication of reasoning and results, and on the appreciation of mathematics. This was reinforced by the State Examinations Commission (2003) Chief Examiner's Report which noted that since the revised curriculum endeavours to reward students for the mathematical knowledge, skills and understanding that they have, it is very important that candidates offer supporting work to outline their thinking throughout the examination paper. A key issue in considering the outcomes of PISA mathematics is the extent of overlap between the revised Junior Certificate mathematics syllabus and PISA.

The last few years have also seen an increased interest in the teaching of mathematics in classrooms, and the role of mathematics more generally in students' education. A video study of mathematics teaching, *Inside Classrooms* (Lyons, Lynch, Close, Sheerin & Boland, 2003), observed 20 mathematics lessons in second year classes in a sample of 10 schools, and found that teaching methods in Ireland were mainly traditional and placed a great deal of emphasis on teacher explanation and questions followed by student practice. It also noted that the methods used were highly focused on preparing students for examinations. Further, the mathematics taught was formal, mainly abstract and generally isolated from real-world contexts. This approach contrasts with the suggested methodology in the *Junior Certificate Mathematics: Guidelines for Teachers* (DES/NCCA 2002) where a variety of interactive strategies are proposed for each mathematics content area. A finding of the ongoing Review of Mathematics in Post-Primary Education is that teaching and learning practices have the greatest influence on students' understanding of mathematics (NCCA, 2006). According to the review, if change is to occur in the mathematical experiences of students, teachers will need to consider ways in which the approaches they use in the class can provide experiences that will engage students more. Conway and Sloane (2005) highlight the most significant trends in mathematics education internationally that might inform the NCCA review of mathematics. One of these trends is realistic mathematics education, which also underpins the PISA assessment of mathematics (see Chapter 2 of this guide).

¹ Words and phrases that are shaded are explained in more detail in the Glossary at the end of this booklet.

Overview of PISA

Students aged 15 were chosen as the target group in PISA as compulsory schooling ends in many countries at this age. In addition to assessing facts and knowledge, PISA assesses students' ability to use mathematical knowledge to solve real-world problems. Therefore, the term 'literacy' is used, since it implies not only knowledge of a domain, but also the ability to apply that knowledge. The main purposes of PISA are:

- to assess **real-world knowledge** and skills and preparedness of students for life-long learning and adult participation in society;
- to provide internationally comparable indicators of student outcomes in key domains at or near the end of compulsory schooling;
- to provide a broad context for countries to interpret such outcomes;
- to determine the nature and extent of associations between school and student factors and achievement outcomes;
- to examine trends in each learning domain over time;
- to provide guidance on developing educational policy.

In all, 41 countries participated in PISA 2003 (Table 1.1). Results were provided for all participating countries except the United Kingdom, which had response rate difficulties.

Table 1.1 Countries Participating in PISA 2003

OECD Countries			Partner Countries
Australia	Iceland	Portugal	Brazil
Austria	Ireland	Slovak Republic	Macao-China
Belgium	Italy	Spain	Hong Kong-China
Canada	Japan	Sweden	Indonesia
Czech Republic	Korea (Rep. of)	Switzerland	Latvia
Denmark	Luxembourg	Turkey	Liechtenstein
Finland	Mexico	United Kingdom	Russian Federation
France	Netherlands	United States	Serbia
Germany	New Zealand		Thailand
Greece	Norway		Tunisia
Hungary	Poland		Uruguay

In addition to tests of mathematics, reading, science and cross-curricular problem solving, student questionnaires were administered to participating students, and school questionnaires to their principal teachers. Topics covered in the student questionnaire include home background, out-of school activities, attitudes towards mathematics, and academic characteristics and behaviours, while the school questionnaire sought information on school structure and composition, school climate, resources, and strategies to promote engagement with mathematics. In Ireland, mathematics teachers in participating schools were asked about qualifications and teaching experience as well as instructional practices, implementation of the revised Junior Certificate mathematics syllabus, and emphasis placed on various aspects of PISA mathematics.

Schools and students that took part in PISA 2003

In Ireland, schools were randomly selected based on size (number of 15-year olds enrolled), type (secondary, community/comprehensive, vocational), and gender composition. Then, up to 35 students within each selected school were chosen randomly. One hundred and forty-one schools (93% of selected schools) agreed to participate. Within these schools, 3,880 students (83% of selected students) participated. Students who did not take part were either absent on the day of testing, or were exempted if, according to international guidelines, they had a functional (physical) disability, a severe general or specific learning disability, or such low proficiency in English that they could not attempt the test. The majority of students who participated in PISA 2003 were in Third year at the time of the study (60.9%), while 2.8% of students were in Second year, 16.7% were in Transition year and 19.6% were in Fifth year.

2 How does PISA assess mathematics?

This chapter describes the PISA 2003 mathematics framework and will help you to interpret the mathematics results presented in Chapter 4. First, mathematical literacy is defined. Then, foundations of the framework are described. Finally, each component of the framework is considered. Sample items that exemplify various aspects of the framework can be found in Chapter 3.

Background to the PISA mathematics framework

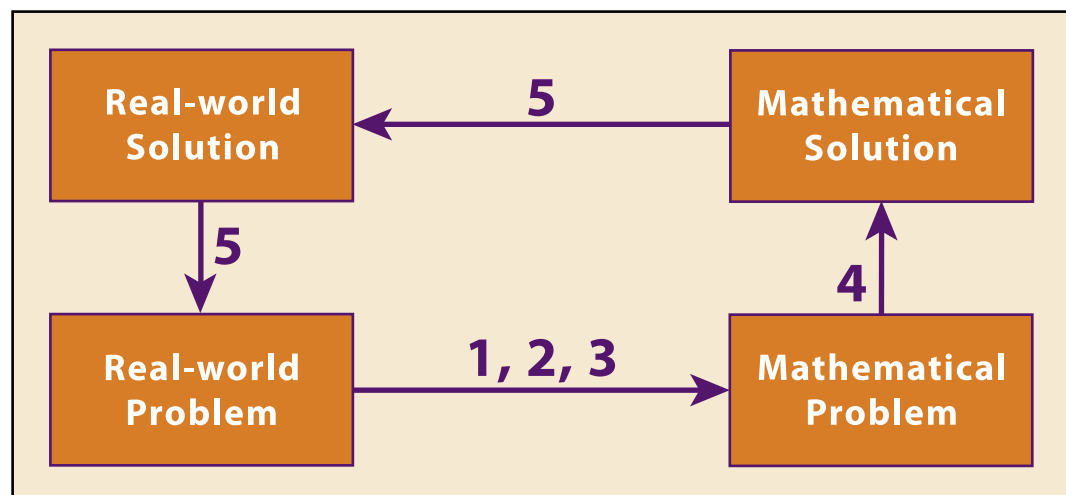
PISA mathematical literacy¹ is defined as ‘an individual’s capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to engage with mathematics in ways that meet the needs of that individual’s life as a constructive, concerned and reflective citizen’ (OECD, 2003, p. 24).

The definition and framework are heavily influenced by the realistic mathematics education (RME) movement, which stresses the importance of solving mathematical problems in real-world settings (e.g., Freudenthal, 1973, 1981). Central to this approach is the notion of mathematising. According to the PISA mathematics framework (OECD, 2003), mathematisation is a five stage process:

1. starting with a problem situated in a real-world context;
2. organising the problem according to mathematical concepts;
3. gradually ‘trimming away the reality’ by making assumptions about which features of the problem are important, and then generalising and formalising the problem;
4. solving the mathematical problem;
5. making sense of the mathematical solution in terms of the real situation.

The process of mathematising is illustrated in Figure 2.1 (numbers indicate the dimensions of mathematisation described above).

Figure 2.1 The Mathematisation Cycle

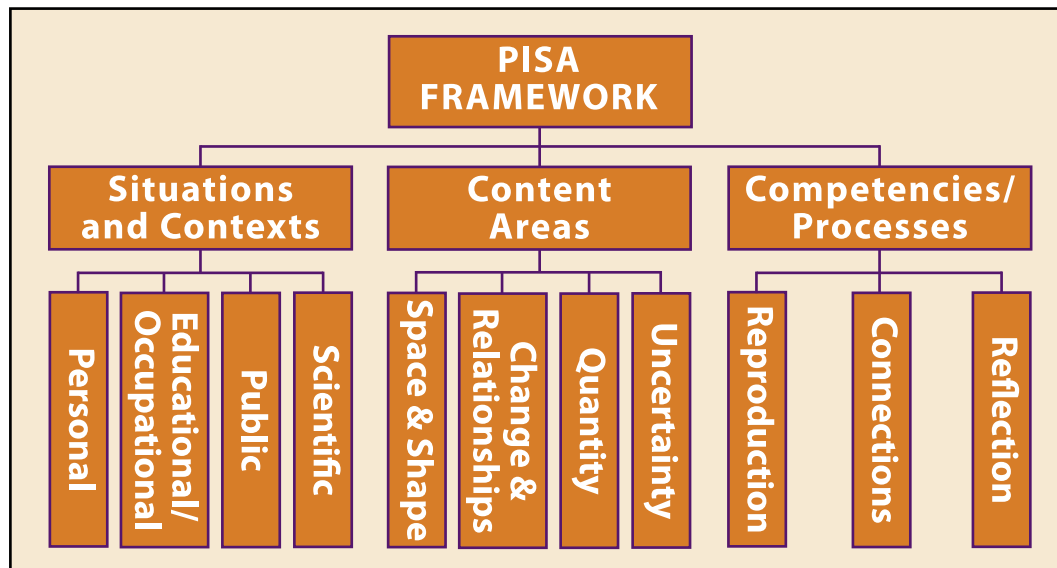


¹ As discussed in Chapter 4, PISA uses the term mathematical literacy to refer to mathematics ability/performance. This report uses the terms ‘mathematical literacy’ and mathematics interchangeably.

Components of the framework

The PISA mathematics framework has three dimensions: (i) situations and contexts; (ii) content; (iii) and competencies (Figure 2.2).

Figure 2.2 Components of the PISA 2003 Mathematics Framework



Mathematics Situations and Contexts

The ability to use and do mathematics in a variety of situations is considered an important part of mathematics education and the type of mathematics employed often depends on the situation in which the problem is presented. In PISA 2003, four categories of mathematical problem situations and contexts are used: personal, educational/occupational, public, and scientific. The situation is the part of the student's world in which the problem arises (e.g., a scientific context). Context reflects the specific setting within that situation (e.g., variation in growth rates).

Mathematics Content Areas

PISA 2003 measured student performance in four areas of mathematics (also called 'overarching ideas'):

- *Space & Shape* – recognising and understanding geometric patterns and identifying such patterns in abstract and real-world representations;
- *Change & Relationships* – recognising relationships between variables and thinking in terms of and about relationships in a variety of forms including symbolic, algebraic, graphical, tabular, and geometric;
- *Quantity* – understanding relative size, recognising numerical patterns and using numbers to represent quantities and quantifiable attributes of real-world objects;
- *Uncertainty* – solving problems relating to data and chance, which correspond to statistics and probability in school mathematics curricula, respectively.

Mathematics Competencies/Processes

PISA identifies eight types of cognitive processes involved in mathematisation – reasoning; argumentation; communication; modelling; problem-posing and -solving; representation; using symbolic, formal and technical language and operations; and use of aids and tools. A mathematical task may involve one or more of these processes at various levels of complexity. In PISA, these processes are represented at different levels of complexity in three broad competency clusters: Reproduction, Connections, and Reflection. Key features of each competency cluster are described in Table 2.1.

Table 2.1 The PISA Competency Clusters

Reproduction Cluster	Connections Cluster	Reflection Cluster
Reproducing representations, definitions and facts	Integrating and connecting across content, situations and representations	Complex problem solving and posing
Interpreting simple, familiar representations	Non-routine problem solving, translation	Reflecting on, and gaining insight into, mathematics
Performing routine computations and procedures	Interpretation of problem situations and mathematical statements	Constructing original mathematical approaches
Solving routine problems	Using multiple well-defined methods	Communicating complex arguments and complex reasoning
	Engaging in simple mathematical reasoning	Using multiple complex methods
		Making generalisations

Source: Adapted from OECD (2003), Figure 1.4, p.49

Classification of Items by Framework Components

Table 2.2 provides a breakdown of PISA 2003 items by situation, content area, and competency cluster. It can be seen that, whereas the four content areas are represented by similar proportion of items, the connections cluster is represented by a greater proportion of items than either the reproduction or reflection cluster. In line with PISA's emphasis on education for citizenship, there are proportionally more items classified as public than personal, educational/occupational or scientific.

Table 2.2 Distribution of PISA 2003 Mathematics Items by Dimensions of the Mathematics Framework

Dimension	Number of Items	Percent of Items
<i>Content Area (Overarching Idea)</i>		
Space & Shape	20	23.5
Change & Relationships	22	25.9
Quantity	23	27.1
Uncertainty	20	23.5
Total	85	100.0
<i>Situation</i>		
Personal	18	21.2
Educational/Occupational	21	24.7
Public	29	34.1
Scientific	17	20.0
Total	85	100.0
<i>Competency Cluster (Process Category)</i>		
Reproduction	26	30.6
Connections	40	47.1
Reflection	19	22.4
Total	85	100.0

It should be noted that, while PISA 2003 mathematics consisted of 85 items, individual students were required to answer only a proportion of those items, as the item pool was distributed over 13 over-lapping test booklets in a **rotated booklet design**.

3 What is a PISA mathematics question like?

This chapter presents some sample mathematics items (questions) from PISA as well as commentaries on the items.

Item types used in PISA

Each student completed a paper-and-pencil test that included a range of mathematics questions. Different item formats (or types of questions) were used to reflect the variety of ways mathematics can be presented and assessed:

- *Traditional multiple-choice items*, in which the student selects a response from among several alternatives [20% of items].
- *Complex multiple-choice items*, in which the student chooses responses for a series of items (e.g., true/false statements) [13%].
- *Closed-constructed response items*, in which the answer is given in numeric or other form, and can be scored against precisely-defined criteria [15%].
- *Short-response items*, in which the student writes a brief answer to a question. Unlike closed-constructed response items, there may be a range of possible correct responses [27%].
- *Open-constructed response items*, in which the student provides a longer written response. There is usually a broad range of possible correct responses. Unlike other item types, the scoring of these questions typically requires significant judgement on the part of trained markers [25%].

PISA 2003 mathematics consisted of 54 units. Each unit consisted of a written description of a problem, associated graphics, and one or more items. A selection of sample items from each mathematics overarching idea is given below. These items were released after the assessment so that they could be used for illustrative purposes. The answer keys accompanying the items illustrate how, for some, there was a single correct answer, while, for others, either partial or full credit was available.

Quantity items

Unit: “Exchange Rate” (*situation: public*)

Mei-Ling from Singapore was preparing to go to South Africa for 3 months as an exchange student. She needed to change some Singapore dollars (SGD) into South African rand (ZAR).

QUESTION 1

Mei-Ling found out that the exchange rate between Singapore dollars and South African rand was: 1 SGD = 4.2 ZAR.

Mei-Ling changed 3000 Singapore dollars into South African rand at this exchange rate.

How much money in South African rand did Mei-Ling get?

Item Type: *Closed constructed response.*

Key: *Full credit:* 12 600 ZAR (unit not required); *no credit:* Other responses, missing.

Process: *Reproduction.* Understand a simple problem and implement a simple algorithm correctly.

PISA Item Difficulty		Item Statistics	% OECD	% Ireland
Scale Score:	406.1	Correct	79.7	83.2
Proficiency Level:	1	Incorrect	13.8	13.4
		Missing	6.6	3.5
		Total	100	100

QUESTION 2

On returning to Singapore after 3 months, Mei-Ling had 3 900 ZAR left. She changed this back to Singapore dollars, noting that the exchange rate had changed to: 1 SGD = 4.0 ZAR.

How much money in Singapore dollars did Mei-Ling get?

Item Type: *Closed constructed response.*

Key: *Full credit:* 975 SGD (unit not required); *no credit:* Other responses, missing.

Process: *Reproduction.* Understand a simple problem and implement a simple algorithm correctly (in reverse).

PISA Item Difficulty		Item Statistics	% OECD	% Ireland
Scale Score:	438.8	Correct	73.9	76.3
Proficiency Level:	2	Incorrect	17.3	18.2
		Missing	8.8	5.5
		Total	100	100

QUESTION 3

During these 3 months the exchange rate had changed from 4.2 to 4.0 ZAR per SGD.

Was it in Mei-Ling's favour that the exchange rate now was 4.0 ZAR instead of 4.2 ZAR, when she changed her South African rand back to Singapore dollars? Give an explanation to support your answer.

Item type: *Open constructed response.*

Key: *Full credit:* 'Yes', with adequate explanation (e.g. Yes, because she received 4.2 ZAR for 1 SGD, and now she has to pay only 4.0 ZAR to get 1 SGD); *no credit:* 'Yes', with no explanation or with inadequate explanation, other responses, missing.

Process: *Reflection.* Identify the relevant mathematics, reduce the task to a problem within the mathematical world, and construct an explanation of the conclusion.

PISA Item Difficulty		Item Statistics	% OECD	% Ireland
Scale Score:	585.3	Correct	40.3	40.8
Proficiency Level:	4	Incorrect	42.3	46.5
		Missing	17.4	12.7
		Total	100	100






The first two questions from this unit belong to the Reproduction cluster. They are both simple problems that require students to link the given information to the required calculation. Students in Ireland performed well on both items (83% and 76% provided correct responses respectively, compared to 80% and 74% on average for OECD countries), although their performance dropped slightly on Question 2, possibly because it requires reverse thinking. Question 3, which belongs to the Reflection cluster, was a more difficult item for students compared to Questions 1 and 2 (41% of students in Ireland answered this question correctly). This item required students to firstly identify the relevant mathematics, compare both answers and then construct an explanation of the conclusion. This may have been a problem for lower-performing students who would be used to more direct questions, and to those who made computational errors on Questions 1 and 2. Unlike the Junior and Leaving Certificate examinations, PISA does not allow students to carry incorrect answers from one part of a question to another. Rather, credit is given only for correct answers. In this respect, PISA does not reward the application of correct mathematical processes to incorrect answers.

You may note that for each item, two pieces of information are presented about the item difficulty in addition to traditional percent correct scores. The first of these is the **(item) scale score**. Items with scale scores below 450 are considered to be easier than average. Those with scale scores between 450 and 550 are deemed to be average in terms of difficulty. Items with a scale score over 550 are considered to be difficult. Therefore, questions 1 and 2 in the Exchange Rate unit are considered to be easier than average, while question 3 is considered to more difficult than average.

The second piece of information on item difficult is the **proficiency level** into which the item falls. Additional information on proficiency levels is given in Chapter 4. For now, it is sufficient to note that items at proficiency levels 1 and 2 can be considered easy, items at level 3 can be considered to have average difficulty, and items at levels 4 and above can be considered to have greater than average difficulty.

Unit: “Skateboard” (*situation: personal*)

Eric is a great skateboard fan. He visits a shop called SKATERS to check some prices. At this shop you can buy a complete board. Or you can buy a deck, a set of 4 wheels, a set of 2 trucks and a set of hardware, and assemble your own board. The prices for the shop's products are:

Product	Price in zeds	
Complete skateboard	82 or 84	
Deck	40, 60 or 65	
One set of 4 Wheels	14 or 36	
One set of 2 Trucks	16	
One set of hardware (bearings, rubber pads, bolts and nuts)	10 or 20	

QUESTION 1

Eric wants to assemble his own skateboard. What is the minimum price and the maximum price in this shop for self-assembled skateboards?

(a) Minimum price: _____ zeds.

(b) Maximum price: _____ zeds.

Item type: *Closed constructed response.*

Key: *Full credit:* Both the minimum (80) and the maximum (137) are correct;

partial credit: Only the minimum (80) is correct, or only the maximum (137) is correct;

no credit: Other responses, missing.

Process: *Reproduction.* Interpret a simple table, find a simple strategy to come up with the maximum and minimum, and use of a routine addition procedure.

PISA Item Difficulty		Item Statistics	% OECD	% Ireland
Scale Score:	463.7 (PC) 496.5 (FC)	Fully Correct	66.7	69.0
		Partially Correct	10.6	8.2
Proficiency Level:	2 (PC) 3 (FC)	Incorrect	18.0	20.8
		Missing	4.7	2.0
		Total	100	100

QUESTION 2

The shop offers three different decks, two different sets of wheels and two different sets of hardware. There is only one choice for a set of trucks. How many different skateboards can Eric construct?

- A 6
- B 8
- C 10
- D 12

Item type: *Traditional multiple choice.*

Key: *Full credit:* D; *no credit:* Other responses, missing.

Process: *Reproduction.* Interpret a text in combination with a table; apply a simple enumeration algorithm accurately.

PISA Item Difficulty		Item Statistics	% OECD	% Ireland
Scale Score:	569.7	Correct	45.5	30.2
Proficiency Level:	4	Incorrect	50.0	66.9
		Missing	4.5	2.9
		Total	100	100

This unit can be considered as presenting archetypal PISA tasks. The introductory scenario involves pictures; moreover, knowledge of the context may well be helpful, though not actually necessary, in addressing the problem. The first question, of Reproduction type, was fairly easy for students in Ireland (69% fully correct), as it was for OECD students in general (67% fully correct). An additional 8% of students in Ireland, and 11% on average across OECD countries had partially correct answers to this question. Students in Ireland did poorly on question 2 (30% correct), compared to the OECD average (46% correct). This is not surprising because the required enumeration algorithm is on the Leaving Certificate rather than the Junior Certificate course, and so would have been unknown to the majority of the group.

UNCERTAINTY ITEMS

Unit: “Earthquake” (*situation: scientific*)

A documentary was broadcast about earthquakes and how often earthquakes occur. It included a discussion about the predictability of earthquakes.

A geologist stated: “In the next twenty years, the chance that an earthquake will occur in Zed City is two out of three.”

QUESTION 1

Which of the following best reflects the meaning of the geologist's statement?

- A $\frac{2}{3} \times 20 = 13.3$, so between 13 and 14 years from now there will be an earthquake in Zed City.
- B $\frac{2}{3}$ is more than $\frac{1}{2}$, so you can be sure there will be an earthquake in Zed City at some time during the next 20 years.
- C The likelihood that there will be an earthquake in Zed City at some time during the next 20 years is higher than the likelihood of no earthquake.
- D You cannot tell what will happen, because nobody can be sure when an earthquake will occur.

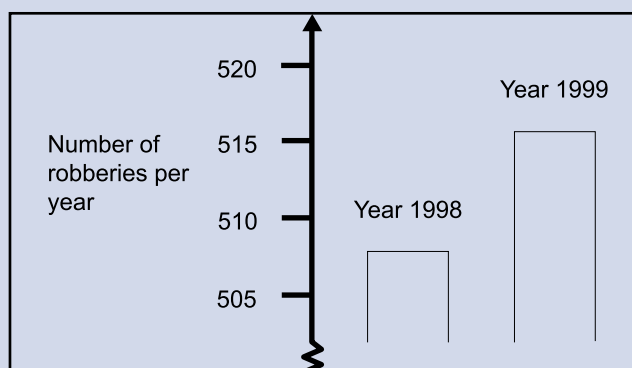
Item type: *Traditional multiple choice*

Key: *Full credit:* C; *no credit:* Other responses, missing.

Process: *Reflection.* Identify the relevant mathematics, and select the conclusion that reflects the meaning of a statement of probability.

PISA Item Difficulty		Item Statistics	% OECD	% Ireland
Scale Score:	557.2	Correct	46.5	51.4
		Incorrect	44.2	41.2
		Missing	9.3	7.4
Proficiency Level:	4	Total	100	100

This unit tests probability, which is not on the Junior Certificate mathematics syllabus (and was not on the Irish Primary School Curriculum at the time at which participating students in PISA 2003 were in primary school). Moreover, the item is classified as being in the Reflection cluster, which tends not to be emphasised on the Junior Certificate syllabus. Nevertheless, students in Ireland (51%) did somewhat better than the OECD average percent correct score (47%).

Unit: “Robberies” (*situation: public*)

QUESTION 1

A TV reporter showed this graph to the viewers and said: “The graph shows that there is a huge increase in the number of robberies from 1998 to 1999.”

Do you consider the reporter’s statement to be a reasonable interpretation of the graph? Give an explanation to support your answer.

Item type: *Open constructed response.*

Key: *Full credit:* “No, not reasonable”. Explanation focuses on the fact that only a small part of the graph is shown (e.g. the entire graph should be displayed);

partial credit: “No, not reasonable”, but explanation lacks detail, or “No, not reasonable”, with correct method but with minor computational errors;

no credit: No, with no, insufficient or incorrect explanation; yes, other responses; missing.

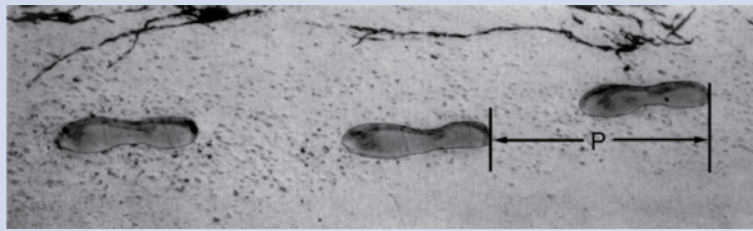
Process: *Connections.* Focus on an increase given by an exact number of robberies in absolute and relative terms; argumentation based on interpretation of data.

PISA Item Difficulty		Item Statistics	% OECD	% Ireland
Scale Score:	576.7 (PC) 694.3 (FC)	Fully Correct	15.4	13.3
		Partially Correct	28.1	36.7
Proficiency Level:	4 (PC) 6 (FC)	Incorrect	41.5	38.1
		Missing	15.0	11.9
		Total	100	100

The single item in this unit was difficult for students, with just 13% in Ireland achieving full credit compared to the OECD average of 15%. On the other hand, 37% of students in Ireland achieved partial credit, compared to an OECD average of 28%. This may reflect the fact that, on the one hand, the material is on the syllabus, but that, on the other hand, the interpretation of misleading graphs has not generally been emphasised in textbooks or examinations. Students due to sit the Junior Certificate examination in 2003 (a few months after taking the PISA tests) or later may have had experience in giving verbal explanations for their answers, as this is a feature of the revised course examined for the first time in 2003; students who sat for the examinations before 2003 would probably have been less accustomed to this.

CHANGE & RELATIONSHIPS ITEMS

Unit: "Walking" (situation: personal)



The picture shows the footprints of a man walking. The pace length P is the distance between the rear of two consecutive footprints. For men, the formula, $n/P = 140$, gives an approximate relationship between n and P where n = number of steps per minute and P = pace length in metres.

QUESTION 1

If the formula applies to Mark's walking and Mark takes 70 steps per minute, what is Mark's pace length? Show your work.

Item type: *Closed constructed response.*

Key: *Full credit:* 0.5 m or 50 cm, $\frac{1}{2}$ (unit not required);

partial credit: $70/p = 140$, $70 = 140p$, or;

no credit: Other responses, missing.

Process: *Reproduction.* Reflect on and realise the embedded mathematics, solve the problem successfully through substitution in a simple formula, and carry out a routine procedure.

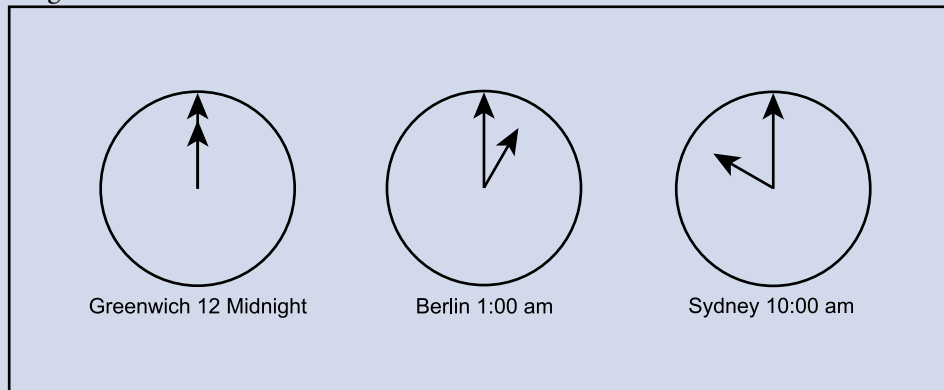
PISA Item Difficulty		Item Statistics	% OECD	% Ireland
Scale Score:	611.0	Fully Correct	36.3	22.9
Proficiency Level:	5	Partially Correct	21.8	34.7
		Incorrect	20.9	28.1
		Missing	21.0	14.3
		Total	100	100

This is an example of an item that has been classified as of Reproduction type but was found difficult; hence, it may illustrate the fact that the relationship between item type and item difficulty is not simple. For students in Ireland the item is not routine. While it tests material on at least the Higher level syllabus, the occurrence of the unknown in the denominator removes it from the realm of often-rehearsed procedures. The percentage of students in Ireland obtaining full credit (23%) is low, but in terms of obtaining at least partial credit, the performance of students in Ireland (35%) is above the corresponding OECD average (22%). The data again illustrate the tendency for students in Ireland to be more ready than average at least to supply an answer, even if incorrect, as just 14% omitted the item, compared with an OECD average of 21%.

Unit: “Internet Relay Chat” (*situation: personal*)

Mark (from Sydney, Australia) and Hans (from Berlin, Germany) often communicate with each other using “chat” on the Internet. They have to log on to the Internet at the same time to be able to chat.

To find a suitable time to chat, Mark looked up a chart of world times and found the following:

**QUESTION 1**

At 7:00 pm in Sydney, what time is it in Berlin?

Answer:

Item type: *Close constructed response.*

Key: *Full credit:* 10 am or 10:00; *no credit:* Other responses, missing.

Process: *Connections.* Establish the time in one time zone, given the time in another.

PISA Item Difficulty		Item Statistics	% OECD	% Ireland
Scale Score:	533.1	Correct	53.7	50.1
Proficiency Level:	3	Incorrect	42.7	48.1
		Missing	3.5	1.8
		Total	100	100

Students in Ireland performed slightly less well on this item compared to the OECD average score (50% versus 54%). One possible reason may be that, although students in Ireland are familiar with using different time zones, some may have been distracted by having information about three time zones, rather than the two required to answer the question. Further, other larger countries (such as the USA or Russia) have multiple time zones and therefore students in these countries may be more familiar with using different time zones.

QUESTION 2

Mark and Hans are not able to chat between 9:00 am and 4:30 pm their local time, as they have to go to school. Also, from 11:00 pm till 7:00 am their local time they won't be able to chat because they will be sleeping.

When would be a good time for Mark and Hans to chat? Write the local times in the table.

Place	Time
Sydney	
Berlin	

Item Type: *Short response.*

Key: *Full credit:* Any time or interval of time satisfying the 9 hours time difference and taken from one of these intervals (e.g. Sydney: 4:30pm – 6:00pm; Berlin: 7:30am – 9:00am);

no credit: Other responses, including one time correct but corresponding time incorrect, missing.

Process: *Reflection.* Satisfy multiple constraints to establish overlap in time between two time zones.

PISA Item Difficulty		Item Statistics	% OECD	% Ireland
Scale Score:	635.9	Correct	28.8	37.2
Proficiency Level:	5	Incorrect	52.1	53.5
		Missing	19.2	9.3
		Total	100	100

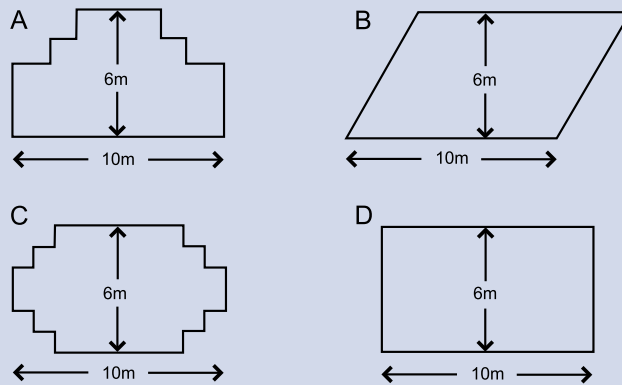
The response patterns for question 2 in particular are of interest. Students in Ireland (37%) performed rather strongly in comparison with the OECD average score (29%), and were much less inclined to omit the item (or at least to provide no answer). This occurred despite the fact that the problem posed in the question is not common in Irish textbooks or examinations, so the students were unlikely to know a routine procedure that would yield a correct answer.

SPACE & SHAPE ITEMS

Unit: "Carpenter" (*situation: educational*)

QUESTION 1

A carpenter has 32 metres of timber and wants to make a border around a vegetable patch. He is considering the following designs for the vegetable patch.



Circle either "Yes" or "No" for each design to indicate whether the vegetable patch can be made with 32 metres of timber.

Vegetable patch design	Using this design, can the vegetable patch be made with 32 metres of timber?
Design A	Yes / No
Design B	Yes / No
Design C	Yes / No
Design D	Yes / No

Item type: *Complex multiple choice.*

Key: *Full credit:* Four correct (yes, no, yes, yes, in that order);

partial credit: Three correct;

no credit: Two or fewer correct; missing.

Process: *Connections.* Use geometrical insight and argumentation skills, and possibly some technical geometrical knowledge.

PISA Item Difficulty		Item Statistics	% OECD	% Ireland
Scale Score:	687.3	Fully Correct	20.0	13.0
Proficiency Level:	6	Partially Correct	30.8	30.9
		Incorrect	46.8	54.6
		Missing	2.5	1.6
		Total	100	100

This was a difficult item for students across OECD countries (20% achieved full credit), and particularly so for students in Ireland (13%). This is a rare example of an item for which the formal study of traditional Euclidean geometry ("technical geometrical knowledge") – more emphasised in the syllabus of Junior and Leaving Certificate levels than in some other countries – might have proved helpful. In particular, such knowledge might have been helpful in identifying the fact that the "slant" sides of the non-rectangular parallelogram are greater than 6m in length; but few students made the required connections. However, skills of visualisation might have proved equally helpful, and these are not greatly featured in the syllabi.

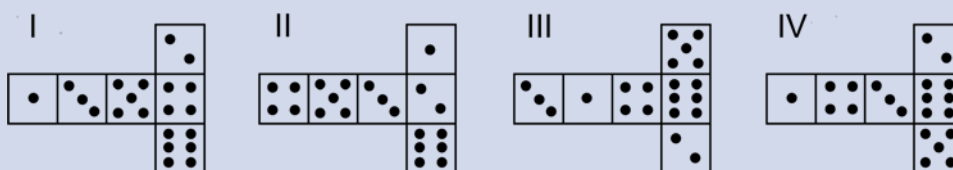
Unit: “Number Cubes” (*situation: personal*)

On the right, there is a picture of two dice. Dice are special number cubes for which the following rule applies: The total number of dots on two opposite faces is always seven. You can make a simple number cube by cutting, folding and gluing cardboard. This can be done in many ways.

**QUESTION 1**

In the figure below you can see four cuttings that can be used to make cubes, with dots on the sides.

Which of the following shapes can be folded together to form a cube that obeys the rule that the sum of opposite faces is 7? For each shape, circle either “Yes” or “No” in the table below.



Shape	Obeys the rule that the sum of opposite faces is 7?
I	Yes / No
II	Yes / No
III	Yes / No
IV	Yes / No

Item type: *Complex multiple choice.*

Key: *Full credit:* No, yes, yes, and no, in that order; *no credit:* Other responses, missing.

Process: *Connections.* Encode and interpret 2-dimensional objects, interpret the connected 3-dimensional object, and check certain basic computational relations.

PISA Item Difficulty		Item Statistics	% OECD	% Ireland
Scale Score:	503.5	Correct	63.0	57.4
Proficiency Level:	3	Incorrect	34.7	40.9
		Missing	2.3	1.7
		Total	100	100

This item requires knowledge of the net of a cube (not on the syllabi at Junior Certificate level) or use of visualisation skills (not emphasised in Ireland, as noted before). The below-average performance on a moderately easy item is thus consistent with expectations based on the Irish curriculum.

The Irish results from PISA 2003 in this area are consistent with the relatively poor Irish performances on “geometry” or “space / shape” elements of previous cross-national studies. In general, in these studies, there has been a tendency for the type of geometry that featured in the Irish syllabi at the time to be under-represented and for the types that did not to be over-represented.

4 How did students in Ireland perform on PISA mathematics?

This chapter provides additional information on the performance of students in Ireland on the overall PISA 2003 mathematics scale and on the four content areas with reference to mean (average) scores and proficiency levels. In addition, differences between low and high achievers, and changes in performance between 2000 and 2003, are examined. Where an achievement difference between groups is said to be **statistically significant**, it can be taken that there is less than a 5% probability that the difference could have arisen by chance.

Overall performance

Each student in the PISA assessment responded to mathematics questions in one of 13 test booklets. As the booklets were linked (each booklet included some items that also appeared in another booklet), it was possible to place each student's performance on the same overall scale. In 2003, the scale was constructed so that the mean student score across OECD countries was 500 points¹, and the **standard deviation** (sd) 100 points². Means and standard deviations on this scale vary across participating countries/regions. Each student's score is based on the difficulty of the tasks (questions) they answer correctly.

Table 4.1 Countries with Mean Scores on Combined Mathematics that Are Significantly Higher than, Not Significantly Different from, and Significantly Lower than Ireland's

Mean Score Significantly Higher than Ireland	Mean Score Not Significantly Different from Ireland	Mean Score Significantly Lower than Ireland	
<i>Hong Kong-Ch</i> (550,▲) Finland (544,▲) Korea (542,▲) Netherlands (538,▲) <i>Liechtenstein</i> (536,▲) Japan (534,▲) Canada (533,▲) Belgium (529,▲) <i>Macao-Ch</i> (527,▲) Switzerland (527,▲) Australia (524,▲) New Zealand (524,▲) Iceland (515,▲)	Czech Rep (517,▲) Denmark (514,▲) France (511,▲) Sweden (509,▲) Austria (506,●) Germany (503,●) [Ireland (503,●)] Slovak Rep (498,●)	Norway (495,▼) Luxembourg (493,▼) Poland (490,▼) Hungary (490,▼) Spain (485,▼) <i>Latvia</i> (483,▼) United States (483,▼) <i>Russian Fed</i> (468,▼) Portugal (466,▼) Italy (466,▼)	Greece (445,▼) <i>Serbia & Monte</i> (437,▼) Turkey (423,▼) <i>Uruguay</i> (422,▼) <i>Thailand</i> (417,▼) Mexico (385,▼) <i>Indonesia</i> (360,▼) <i>Tunisia</i> (359,▼) <i>Brazil</i> (356,▼)
Non-OECD ('partner') countries in italics; (▲) = mean score above OECD average; (●) = mean score not significantly different from OECD average; (▼) = mean score significantly lower than OECD average			

¹ Each OECD country contributed the same number of students for the purpose of calculating this mean.

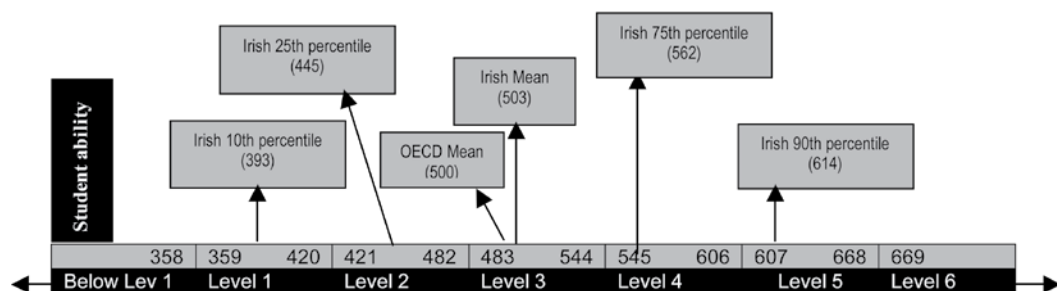
² OECD country average mean scores and standard deviations on the four mathematics subscales are broadly similar: Change & Relationships – mean = 499, sd = 109; Space & Shape – mean = 496, sd = 110; Quantity – mean = 501, sd = 102; and Uncertainty – mean = 502, sd = 99.

Ireland achieved an overall mean score of 502.8, and ranked 17th of 29 OECD countries, and 20th of 40 participating countries (Table 4.1). Twelve countries (including Hong Kong-China, Finland and Korea) had mean scores that are significantly higher than Ireland. Eight countries, including Denmark, Sweden, France and Germany, had mean scores that are not significantly different from Ireland. Norway, the United States, and the Russian Federation were among the countries with mean scores that are significantly lower than Ireland.

Ireland's mean score (502.8) is not significantly different from the OECD country average of 500. Other countries with mean scores that are not significantly different from the OECD country average are Austria, Germany and the Slovak Republic.

Figure 4.1 shows the scores of students in Ireland at key benchmarks on the PISA combined mathematics scale. The score of students in Ireland at the 10th percentile is 393. Students at this point did as well as or better than 10% of students nationally, and less well than 90%. The score of students in Ireland at the 90th percentile is 614. Students at this point did as well as, or better than, 90% of students nationally. Students' scores are also described in terms of proficiency levels (what students at different levels of ability can do). These levels are described in more detail in the mathematics proficiency scales section of this chapter. The scale on Figure 4.1 also shows score point intervals between six mathematics proficiency levels. For example, Level 1 extends from 359 points to 420, while Level 5 extends from 607 to 688.

Figure 4.1 The PISA 2003 Combined Mathematics Scale: Scores of Students in Ireland at Key Markers



Differences between high and low achievers

The gap between the best and poorest performing students within a country, as well as between countries and the corresponding OECD average, can be observed by examining the scores of students at the 10th and 90th percentile ranks. In Ireland, students scoring at the 10th percentile on the combined mathematics scale achieved a score of 393, which is 34 points higher than the corresponding OECD country average. It is also higher than the scores of students at the same benchmark in some countries with mean scores similar to Ireland, including Germany (363) and Norway (376), suggesting a smaller 'tail' of low achievers in the Irish distribution. Students in Ireland at the 90th percentile achieved a score (614), which is lower (by 14 points) than the corresponding OECD country average, and lower than the scores of high achievers in some countries with similar mean scores to Ireland, including Germany (632) and Sweden (631). This suggests students in Ireland scoring at the 90th percentile in particular are underperforming relative to their counterparts at the same benchmark in other countries with similar overall performance. In general, the difference between high and low achievers in Ireland (221 points) is smaller than the OECD average difference (259), indicating a relatively narrow spread in achievement (a finding also observed when performance on the proficiency levels is considered).

Mathematics proficiency scales

A feature of item response theory scaling, which was used with the PISA data, is that test item difficulties and student scores can be placed on the same scale. This can provide insights into what students at different levels of ability can do. The combined mathematics scale was divided into six levels of proficiency, each characterised by different levels of skills and knowledge. The difference between one level and the next is about 62 score points. The descriptions on Table 4.2 are based on analyses of the content and processes underlying items at each proficiency level.

Table 4.2 Summary Descriptions of Proficiency Levels on the Combined Mathematics Scale, and Percentages of Irish and OECD Students Achieving Each Level

Level	Summary Description
Level 6	Conceptualise, generalise, and utilise information based on investigations and modelling of complex problem situations; link different information sources and representations and flexibly translate among them; demonstrate advanced mathematical thinking and reasoning, and apply this insight along with a mastery of symbolic and formal mathematical operations and relationships to develop new approaches and strategies for attacking novel situations; formulate and precisely communicate actions and reflections regarding findings, interpretations, arguments, and the appropriateness of these to the original situations.
Level 5	Develop and work with models for complex situations, identifying constraints and specifying assumptions; select, compare, and evaluate appropriate problem-solving strategies for dealing with complex problems; work strategically using broad, well-developed thinking and reasoning skills, appropriate linked representations, symbolic and formal characterisations, and insight pertaining to these situations; and reflect on their actions and formulate and communicate their interpretations and reasoning.
Level 4	Work effectively with explicit models for complex concrete situations that may involve constraints or call for making assumptions; select and integrate different representations, including symbolic ones, linking them directly to aspects of real-world situations; utilise well-developed skills and reason flexibly, with some insight, in these contexts; and construct and communicate explanations based on own interpretations, arguments, and actions.
Level 3	Execute clearly described procedures, including those that require sequential decisions; select and apply simple problem-solving strategies; interpret and use representations based on different information sources and reason directly from them and develop short communications reporting interpretations, results and reasoning.
Level 2	Interpret and recognise situations in contexts that require no more than direct inference, extract relevant information from a single source and make use of a single representational mode; employ basic algorithms, formulae, procedures, or conventions, and demonstrate direct reasoning and make literal interpretations of the results.
Level 1	Complete tasks involving familiar contexts where all relevant information is present and the questions are clearly defined; identify information and carry out routine procedures according to direct instructions in explicit situations; and perform actions that are obvious and follow immediately from the given stimuli.
Below Level 1	Has less than .50 chance of responding correctly to Level 1 tasks. Mathematics skills not assessed by PISA.

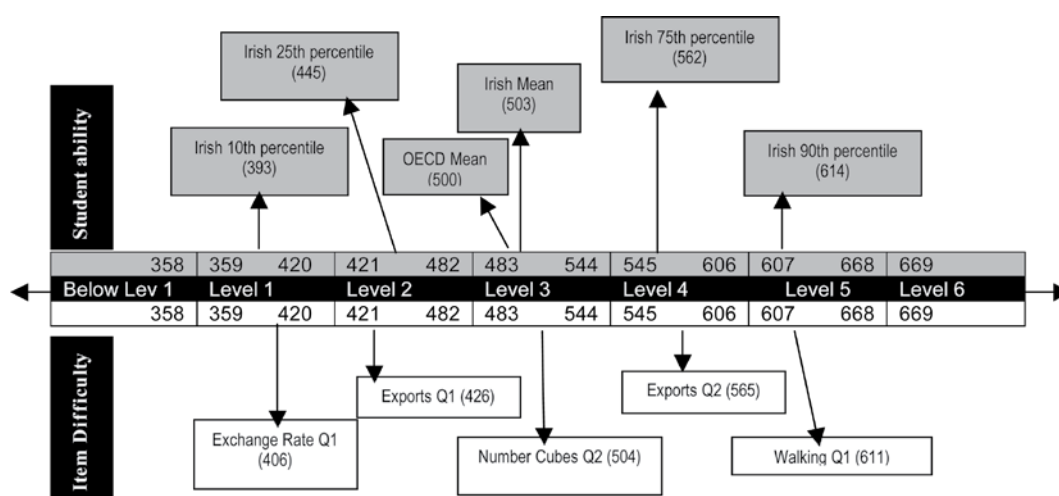
Source: Cosgrove et al. (2005), Table 3.11.

The PISA proficiency levels were defined in such a way that all students at a given level are expected to respond correctly to at least half of the items they attempt at that level. Further, they are expected to respond correctly to fewer than one-half of items at higher levels, and more than one-half of items at lower levels.

Level 6, the highest level, has no ceiling. This means that some high-achieving students have an ability that is higher than the most difficult PISA mathematics items and are likely to get most of the PISA mathematics items they attempt correct. On the other hand, students with a score below Level 1 are unlikely to succeed at even the easiest PISA mathematics items.

In addition to student scores at key benchmarks, Figure 4.2 shows item difficulties for selected mathematics items described in Chapter 3. The second item from the unit 'Number Cubes' is located at Level 3 on the proficiency scale. It has a difficulty (504), which is close to the mean score for students in Ireland (503). The first item from the unit 'Walking' is located at Level 6. It has an item difficulty (611) that is close to the score of students in Ireland at the 90th percentile (614). Question 1 in the 'Exchange Rate' unit (also described in Chapter 3) is at Level 1 on the proficiency scale, and has an item difficulty of 406. Since this is a full standard deviation below the mean score for students in Ireland, it can be considered quite easy.

Figure 4.2 The PISA 2003 Combined Mathematics Scale: Cut-points for Proficiency Levels, Scores of Students in Ireland at Key Markers, and Difficulties of Selected Items



In Ireland, 11% of students scored at the highest mathematics proficiency levels (Levels 5 and 6 combined) (Table 4.3). The corresponding OECD average was 15%. This indicates that there are fewer higher-achieving students at these levels in Ireland than the average across OECD countries. Indeed, 21 countries had more students than Ireland scoring at Levels 5 and 6, including Hong-Kong (31%), Finland (24%) and Canada (20%). Seventeen percent of students in Ireland scored at the lowest levels (Level 1 and below), compared to an OECD average of 21%. Hence, there are fewer very low achievers in Ireland than there are on average across OECD countries. The observation that 72% of students in Ireland score at Levels 2, 3 and 4, compared to 64% of students at these levels on average across OECD countries, indicates that students in Ireland tend to 'bunch up' at the average proficiency levels, with relatively few students at the extremes (Level 1 and below or Levels 5 and 6).

Table 4.3 Percentages of Students in Ireland, and OECD Average Percentages, Scoring at Each Proficiency Level on PISA Combined Mathematics

Proficiency Level	Ireland	OECD Average
Level 6 (highest)	2.2	4.0
Level 5	9.1	10.6
Level 4	20.2	19.1
Level 3	28.0	23.7
Level 2	23.6	21.1
Level 1	12.1	13.2
Below Level 1 (lowest)	4.7	8.2
Totals	100.0	100.0

Performance on the four mathematics content areas

The performance of students in Ireland is significantly above the OECD average on the Change & Relationships and Uncertainty content scales, while Ireland's performance is significantly lower than the OECD average on the Space & Shape scale, and does not differ significantly from the OECD average on the Quantity scale (Table 4.4). Of the 29 OECD countries for which results were available, Ireland ranked 10th on the Uncertainty scale, 15th on the Change & Relationships subscale, 18th on the Quantity subscale, and 23rd on the Space & Shape subscale.

Table 4.4 Mean Scores and Standard Deviations on the Mathematics Content Scales—Ireland and OECD

Country	Space & Shape		Change & Relationships		Quantity		Uncertainty	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Ireland	476.2 ^a	94.5	506.0 ^b	87.5	501.7 ^c	88.2	517.2 ^b	88.8
OECD	496.3	110.1	498.8	109.3	500.7	102.3	502.0	98.6
^a significantly below the OECD average ^b significantly higher than the OECD average ^c not significantly different from OECD average								

Proficiency levels were also developed for the mathematics content scales, using the same cut-off points as for the combined mathematics scale. There were fewer lower achievers (at Level 1 or below) in Ireland compared to the OECD country average on all but one scale – Space & Shape (Table 4.5). Similarly, fewer students in Ireland attained the highest levels of proficiency (Levels 5 and 6) on any of the content scales, with the exception of Uncertainty, where 16% achieved Levels 5 and 6 (OECD average = 15%).

Table 4.5 Percentage of Students Achieving at Each Proficiency Level for Each of the Mathematics Content Scales—Ireland and OECD

Scale		<Level 1	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Space & Shape	Ireland	10.7	16.9	25.4	23.0	15.4	6.8	1.8
	OECD	10.6	14.2	20.4	21.5	17.2	10.4	5.8
Change & Relationships	Ireland	5.1	11.2	22.6	27.0	21.6	10.2	2.3
	OECD	10.2	13.0	19.8	22.0	18.5	11.1	5.3
Quantity	Ireland	5.6	12.3	23.0	26.9	20.6	9.5	2.2
	OECD	8.8	12.5	20.1	23.7	19.9	11.0	4.0
Uncertainty	Ireland	3.6	10.2	21.2	26.5	22.0	12.4	4.0
	OECD	7.4	13.3	21.5	23.8	19.2	10.6	4.2

Performance in PISA 2003 mathematics compared to 2000

The 2000 assessment included items in two of the four mathematics content areas (Space & Shape, and Change & Relationships) tested in the 2003 assessment. Hence, performance in 2000 and 2003 can only be compared on these two subscales. There was no significant change in the mean scores for students in Ireland on either content scale between 2000 and 2003. Although the OECD average score on the Space & Shape scale did not change between the two years, eight countries, including Belgium and the Czech Republic, registered a significant improvement in 2003, while two – Iceland and Mexico – had mean scores that were significantly lower. The OECD average score improved to a significant extent on the Change & Relationships scale. There was a significant increase in mean performance in 13 countries in 2003, including Korea and Germany, and a significant decrease in just one (Thailand).

Chapter Highlights

- In PISA 2003, students in Ireland achieved a mean overall mathematics score of 503, which is not statistically different from the OECD country average of 500.
- Students in Ireland ranked 17th of 29 OECD countries and 20th among 40 participating countries on the combined mathematics scale.
- Fewer students in Ireland (11%) achieved at the highest mathematics proficiency levels (Levels 5 and 6), compared to the OECD average (15%), indicating that there are fewer very high achievers in Ireland, compared with the OECD average.
- Fewer students in Ireland (17%) achieved at the lowest proficiency levels (at or below Level 1) than the OECD average (21%), indicating that there are fewer very low achievers in Ireland compared with the OECD average.
- Students in Ireland achieved mean scores that were above the OECD average on two mathematics content scales (Change & Relationships and Uncertainty), at about the OECD average on one (Quantity), and below the OECD average on one (Space & Shape).

5 How does PISA mathematics compare with Junior Certificate mathematics?

In this chapter, links between PISA mathematics and Junior Certificate mathematics are considered. First, the content and processes underpinning the two assessments are compared. Then, the performance of students who participated in PISA mathematics in 2003, and sat the Junior Certificate mathematics examination in 2002 or 2003, are compared.

Comparing the content and processes of PISA and Junior Certificate mathematics

Objectives of PISA and Junior Certificate Mathematics

Many of the objectives of the Junior Certificate mathematics syllabus are reflected in the PISA mathematics framework. For example, the Junior Certificate mathematics objectives of recalling mathematical facts and establishing competencies needed for mathematics activities (instrumental understanding) are consistent with the assumption underlying the PISA framework that, by age 15, students will have mastered basic mathematics skills. Further, the Junior Certificate objective of developing relational understanding is consistent with the PISA view that students need a conceptual understanding of procedures to know which to apply to solve a real-world mathematical problem.

Although many of the Junior Certificate objectives compare well with the aims of PISA, not all of them are assessed in the Junior Certificate examination. For example, PISA emphasises real-world knowledge and skills, and therefore the ability to solve problems in novel contexts is an important prerequisite for many of the items. However, the only Junior Certificate objective addressing this skill (analysis of information, including that presented in unfamiliar contexts) is not actually assessed in the Junior Certificate examination and therefore is likely to receive less emphasis in instruction than those objectives that are assessed. Other objectives that are not assessed in the Junior Certificate examination relate to the ability to create mathematics and development of an appreciation of mathematics. Yet these are consistent with PISA's emphasis on the importance of fostering an interest in and appreciation of mathematics – valuable educational outcomes in themselves.

Relationship between the PISA Items and Junior Certificate Mathematics

To acquire a better understanding of the links between PISA and Junior Certificate mathematics, curriculum experts in Ireland (all experienced teachers of mathematics) were asked to rate the expected familiarity of each PISA mathematics item for a typical third-year student. Each item received nine ratings, one for each of three aspects (concept, context of application, format) at each of three syllabus levels (Higher, Ordinary, Foundation). Ratings ranged from 1 ('not familiar') to 3 ('very familiar').

The concepts underlying approximately two-thirds of items were rated as being somewhat or very familiar to students at Higher and Ordinary levels while just under half of the items were rated in this way for students at Foundation level (Table 5.1). On the other hand, the contexts in which the mathematics problems were presented (usually real-world situations)

and the item formats (often multiple-choice) were judged to be mostly unfamiliar to students in Ireland at all three syllabus levels. Of course, the fact that students are not asked to attempt very many multiple-choice items in Junior Certificate mathematics does not imply that they cannot attempt such items, as the multiple-choice format may well be familiar from other contexts (e.g. standardised tests).

Table 5.1 PISA 2003 Mathematics Curriculum Familiarity Ratings, by Junior Certificate Level

	Not familiar %	Somewhat familiar %	Very familiar %
Concept			
Higher	30.6	24.7	44.7
Ordinary	35.3	29.4	35.3
Foundation	51.8	25.9	22.4
Context			
Higher	65.9	22.4	11.8
Ordinary	70.6	20.0	9.4
Foundation	80.0	16.5	3.5
Format			
Higher	62.4	24.7	12.9
Ordinary	72.9	20.0	7.1
Foundation	83.5	14.1	2.4
Note. Ratings on these scales are made considering the typical third-year student at each syllabus level.			

PISA items were also rated by three expert raters in terms of the Junior Certificate mathematics syllabus area into which they mainly fell. Table 5.2 shows that 29% of PISA items could not be located in the Higher level syllabus, 33% could not be located in the Ordinary level syllabus, and 49% could not be located in the Foundation level syllabus.

Table 5.2 PISA 2003 Mathematics Items Cross-tabulated with Junior Certificate Mathematics Areas

Junior Certificate Math Strand Area	Percent of PISA Items Located in:		
	Higher	Ordinary	Foundation
Not on Junior Cycle syllabus	28.6	33.0	49.4
Number systems	8.8	9.9	9.0
Applied arithmetic and measure	33.0	31.9	25.8
Algebra	5.5	4.4	1.1
Statistics	19.8	17.6	14.6
Functions and graphs	4.4	3.3	0.0
Sets	0.0	0.0	0.0
Geometry	0.0	0.0	0.0
Trigonometry	0.0	0.0	0.0 (n/a)
Total	100.0	100.0	100.0
N = 85 PISA items. However, 6 items were identified as being located in two Junior Certificate mathematics areas at Higher and Ordinary levels, and 4 items in the case of Foundation level. Hence, totals are 91, 91 and 89 respectively. Source: Cosgrove et al. (2005), Table 6.11.			

It is noteworthy that none of the PISA Space & Shape items were classified as being in the Junior Certificate Geometry strand at any syllabus level. This reflects differences between PISA Space & Shape, which focuses more on visualization skills (recall the 'cube' example), and Junior Certificate Geometry, where the emphasis tends to be on more traditional Euclidian geometry. In fact, at Higher and Ordinary levels, 13 of the 20 PISA Space & Shape items were classified as being in the Junior Certificate area of Applied Arithmetic & Measure. Most or all of the remaining PISA Space & Shape items (depending on level) were deemed not to be on the syllabus. This suggests that, in doing PISA, students in Ireland do not get a direct opportunity to demonstrate what they had learned in Junior Certificate Geometry.

It is also apparent from Table 5.2 that relatively few PISA items fell directly into the Junior Certificate area of Algebra. Given the very heavy emphasis on Algebra in the syllabus and in the Junior Certificate mathematics examination, it can be argued that students in Ireland did not get an opportunity to demonstrate their knowledge in this area on PISA.

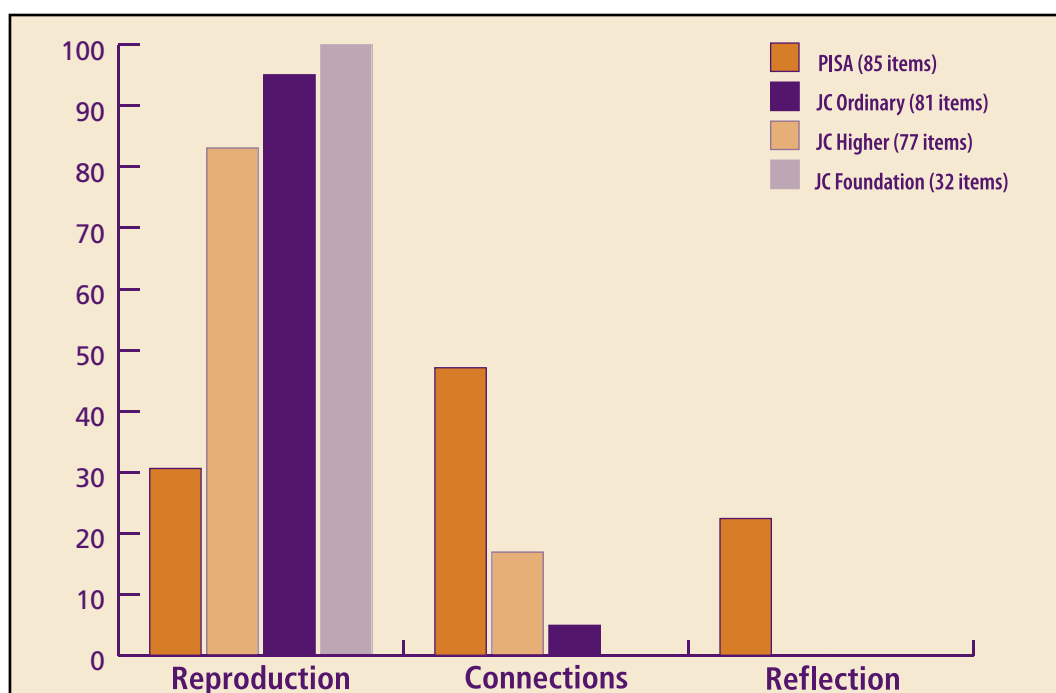
Correlations between Expected Familiarity with PISA Items and Overall PISA Scores

Taking into account the particular PISA mathematics items to which students responded to, along with the level at which they took the Junior Certificate mathematics examination, familiarity scores for each aspect were computed for each student. The correlation between the aggregated context ratings and the performance of students on the PISA combined mathematics score is weak to moderate (.21), while the correlations between the aggregated concept and format familiarity ratings and performance are moderate (.28 for format, and .37 for concepts). These correlations indicate that students typically did better on items in which underlying concepts, formats and, contexts were expected to be familiar to them, than on items for which these aspects were not expected to be as familiar. The correlation between context and performance is consistent with Close's (2006) conclusion that students in Ireland may have been particularly disadvantaged by their lack of familiarity with the contexts in which many of the PISA items were presented.

Examining Junior Certificate Mathematics Items through the Lens of PISA

A study by Close and Oldham (2005) examined the extent to which specific mathematics items on the 2003 Junior Certificate Mathematics Examination mapped onto the PISA mathematics framework. Here, we look at the categorisation of Junior Certificate items into PISA competency clusters. Figure 5.1 shows that over 80% of items in the Higher level examination, over 90% in the Ordinary level examination, and all of the items in the Foundation level examination fell into the PISA Reproduction cluster. This compares with 30% of PISA items that were categorised as Reproduction. The figure also indicates that, whereas over 50% of PISA items fell into the 'Connections' cluster, this was so for 17% of Higher level items, 5% of Ordinary level items, and no Foundation level items. Finally, although over 20% of PISA mathematics items were categorised as 'Reflect' (indicating that they called on higher-level thinking processes, including communicating the reasoning underlying the solution to a problem), none of the Junior Certificate mathematics items was rated in this way. This suggests that Junior Certificate students might have been unprepared for PISA items requiring reflection.

Figure 5.1 Percentages of PISA and Junior Certificate Mathematics Examination Items, by PISA Competency Clusters.



Source: Close & Oldham, 2005, Figure 2.

Comparing performance on PISA mathematics with performance on the Junior Certificate mathematics examination

Junior Certificate Performance Scale

Almost 94% of students who took the PISA assessment of mathematics sat the Junior Certificate mathematics examination in 2002 or 2003, and their grades on the examination were compared to their performance on the PISA assessment. Grades on the Junior Certificate mathematics examination were converted to a 12-point Junior Certificate Performance Scale such that a 12 corresponds to an A at Higher level and a 1 corresponds to an F at Foundation level (Table 5.3). The correlation between students' scores on PISA mathematics and their Junior Certificate Performance Scale scores in mathematics is .75. Among the PISA subscales, the correlation between performance on Junior Certificate mathematics and the Space & Shape scale is weakest at .68, while it is .73 for Quantity and .74 for both the Change & Relationship and Uncertainty scales. These coefficients show broad overlap between performance on the two assessments, with students who did well on one generally doing well on the other. This pattern is interesting in light of the differences in content between the two assessments discussed earlier.

Table 5.3 Junior Certificate Performance Scale

Junior Certificate Performance Scale Score												
Syllabus Level	12	11	10	9	8	7	6	5	4	3	2	1
Higher	A	B	C	D	E	F						
Ordinary				A	B	C	D	E	F			
Foundation							A	B	C	D	E	F

Mean PISA Scores at Each Syllabus Level

Students' performance in PISA mathematics was compared with their performance on the Junior Certificate examination at each of the three syllabus levels. There are large average differences in the PISA mean scores of students taking mathematics at Higher and Ordinary levels (94 points), and those taking mathematics at Ordinary and Foundations levels (84 points). The performance difference between Higher and Foundation level students is smallest for the Space & Shape subscale (174.6 score points) and largest for the Change & Relationships subscale (186.8 score points).

Percentages of Students at Each PISA Proficiency Level, by Junior Certificate Examination Syllabus Level

Mean scores on PISA can also be interpreted in terms of the PISA mathematics proficiency levels, described in Chapter 4. The mean score of Higher level students (563.0) is at Level 4. Ordinary level students have a mean score (469.1) which is at Level 2, and Foundation level students have a mean score (385.4) which is at Level 1.

Table 5.4 shows the percentages of students at each PISA proficiency level classified by the syllabus level at which they took the Junior Certificate mathematics examination. As can be seen from the table, one third of Foundation level students scored below Level 1, indicating that they did not demonstrate even the most basic skills associated with PISA mathematics, while no student taking Foundation level demonstrated a proficiency higher than Level 3. Just over one-fifth of students at Ordinary level are at or below Level 1, while less than half of students at Ordinary level are at Level 3 or higher. If one accepts the OECD (2004) specification of Level 2 as a basic minimum that students need to achieve to meet their future needs in education and the world of work, it is a matter of concern that relatively large proportions of students taking Ordinary and Foundation levels achieved below this benchmark.

Table 5.4 Percentages of Students in Ireland at Each PISA Mathematics Proficiency Level, Classified by Junior Certificate Mathematics Examination Level (2002 and 2003).

Syllabus Level	% Below Level 1	% at Level 1	% at Level 2	% at Level 3	% at Level 4	% at Level 5	% at Level 6	Total
Higher	0.3	1.2	9.0	28.8	35.8	19.7	5.2	100
Ordinary	4.1	17.8	36.2	30.4	9.9	1.5	0.1	100
Foundation	33.4	38.5	22.5	5.5	0.0	0.0	0.0	100
Number of students at Higher = 1651; Ordinary = 1941; Foundation = 265; Missing = 24.								

Chapter Highlights

- The majority of concepts underlying PISA mathematics items were considered by curriculum experts in Ireland to be 'somewhat familiar' or 'very familiar' to students following Higher and Ordinary level mathematics.
- The majority of the 'real-world' contexts in which PISA mathematics items were embedded were judged by experts to be unfamiliar to students in Ireland.
- None of the Space & Shape items on PISA mathematics were located in the Geometry strand of the Junior Certificate mathematics syllabus, indicating that students in Ireland may not have had an opportunity to demonstrate their knowledge of Junior Certificate Geometry on the PISA mathematics assessment.
- Just 5% of PISA items were located in the Algebra strand of the Junior Certificate mathematics syllabus, again highlighting differences between PISA mathematics and Junior Certificate mathematics.
- Whereas the majority of PISA items required higher-level processing such as 'Connecting' and 'Reflecting', the majority of items on the 2003 Junior Certificate mathematics examination required students to 'Reproduce'. This suggests that students in Ireland may not have had sufficient opportunity to engage in higher-level mathematics processing required by an assessment such as PISA.
- One-fifth of students taking the Ordinary level examination and two-thirds taking the Foundation level examination in 2002 or 2003 did not reach the minimum level that, according to the OECD, students require to meet their future needs in education and in the work place.

6 How do student and school characteristics relate to performance on PISA mathematics?

This chapter examines **variables** associated with performance on PISA mathematics. The first section examines associations between student characteristics and performance, while the second examines school characteristics and performance. The third section looks at differences in performance between schools (between-school variance), while the final section describes a multi-level model of achievement on PISA mathematics, which seeks to explain the simultaneous contributions of selected student and school characteristics.

Student characteristics

Student Gender

Male students significantly outperformed females on the combined mathematics scale in 21 of 29 OECD countries, including Ireland. The difference between males and females in Ireland was moderate (15 points), and slightly larger than the OECD country average (11). Iceland is the only country in which females' significantly outperformed males.

Males in Ireland also scored significantly higher than females on all four mathematics content scales (Table 6.1). The difference is greatest for the Space & Shape scale (26 points) and smallest for Quantity (9 points).

Table 6.1 Mean Scores of Students in Ireland on the PISA Mathematics Content Scales, by Gender

	% of Students	Space/ Shape	Change/ Rel.	Quantity	Uncertainty
Males	50.4	488.9	512.2	506.1	524.9
Females	49.6	463.4	499.6	497.2	509.4
All available	100.0	476.2	506.0	501.7	517.2

More males than females achieved proficiency Level 5 or above on the combined mathematics scale (13.7% compared to 9.0%) and more females than males had a proficiency level at or below Level 1 (18.7% compared to 15.0%).

Student Home Background

Parental occupation was used as a measure of student socioeconomic status (SES). All students were asked to indicate their mother's and father's main occupations, and to give a brief description of the nature of each parent's work. These responses were categorised according to the International Socioeconomic Index (ISEI) system and then grouped as high, medium, or low SES (Table 6.2). Students with high SES scores significantly outperformed students with medium and low scores in mathematics, with the largest difference between high and low SES groups (62 points).

Table 6.2 Mean Combined Mathematics Scores of Students in Ireland , by Socioeconomic Group

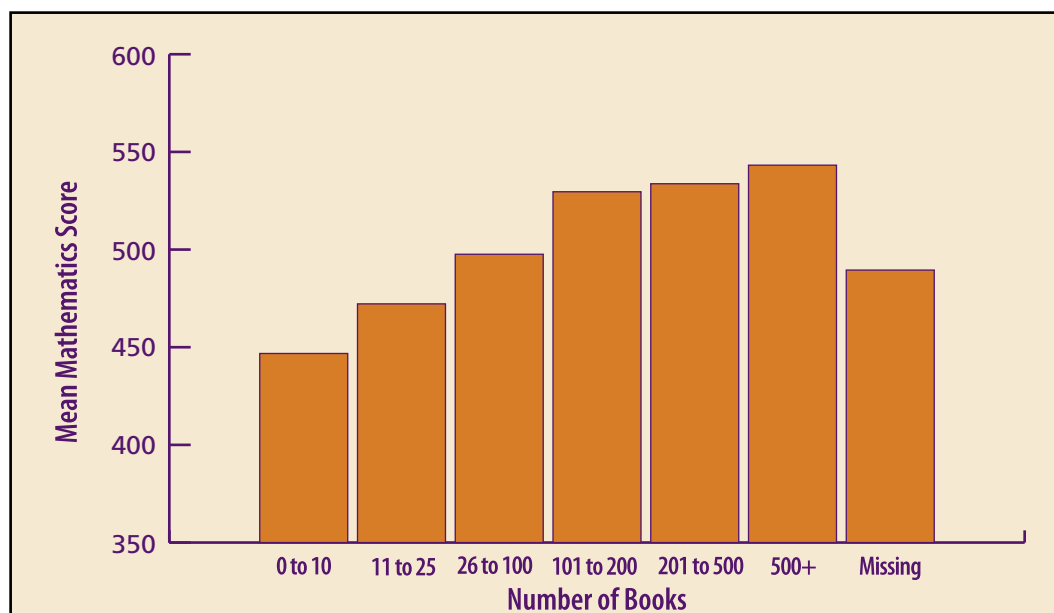
SES Level	Percent of Students	Mean Mathematics Score
High	31.1	535.7
Medium	33.6	506.1
Low	31.0	473.5
No response	4.3	452.0
All available cases	95.7	505.1

The impact of individual socioeconomic background on the achievement of students in Ireland is not significantly different from the OECD average impact (OECD, 2004, Figure 4.9).

Family Structure and Home Educational Resources

Students were asked to provide information on household composition (i.e., who usually lived at home with them). Students who lived in lone-parent families obtained a mean score that is considerably lower (by 34 points) than the mean score of students who lived in a two-parent household. Students who had no siblings achieved a mean score that is moderately lower (by 21 points) than the mean score of students with one sibling. Students with one sibling did not differ in mean performance from students with two siblings, but they outperformed students with 3 siblings (by 23 points), and students with 4 or more (by 39 points).

Students indicated whether they had a desk for study at home, a quiet place to study, and books to help them with schoolwork. Students with none or one of these were categorised as having 'low' home educational resources, students with any two as having medium resources, and students with all three as having high resources. Students with low home educational resources had a mean score in mathematics that is lower (by 24 points) than the mean of students with medium resources, and lower (by 44 points) than the mean of students with high resources. Students also provided information about the number of books in their homes. Higher achievement scores were associated with having access to more books at home (Figure 6.1). The 10% of students who reported having 10 or fewer books scored almost 100 points lower than students with 500 or more books.

Figure 6.1 Mean Mathematics Scores, by Number of Books in Students' Homes

Student Academic Characteristics

Homework and Study

Students reported spending an average of 7.7 hours on homework/study per week (including weekends) across all subjects and 2.8 hours on mathematics. Students were categorised into low, medium, and high groups according to the amount of time they typically spent on mathematics homework. The mean score of students who spent low amounts of time on mathematics homework (491.0 points) is significantly lower than the mean scores of students who spent medium (519.7) or high (512.0) amounts, while the difference between the medium and high groups is not statistically significant. Similar results were found for the total amount of time spent on homework/study across all subjects.

Absence from School

Just under 10% of students indicated that they had been absent for three or more days in the two weeks prior to the PISA assessment, while a majority (58%) reported attending every day over the same period. Students with full attendance significantly outperformed students who were absent for 1 or 2 days by almost 20 points (514.7 compared to 495.1) and students who were absent for 3 or more days by 50 points (514.7 compared to 465.0).

Calculator Use

Calculator use in PISA 2003 was optional. Almost 80% of all students reported using a calculator on the mathematics items. Students who reported using a calculator during the assessment had a significantly higher mathematics score (by 20 points) than students who did not use a calculator. This is consistent with findings from a recent study of calculators in the Junior Cycle mathematics curriculum and examinations, which indicates that calculator access improves performance on complex real-world problem-solving items (Close et al., 2003).

Grade Level

Participants in PISA in Ireland were spread over 4 grade levels – Second year (2.8%), Third year (60.9%), Fourth/Transition year (16.7%) and Fifth year (19.6%). Students in Fifth year achieved a mean score (515.5) that is significantly lower than the mean score of students in Fourth year (542.9). Students in Third year (492.3) outperformed students in Second year (406.8), but did less well than students in Fourth and Fifth years.

Students' Self-Efficacy in, and Anxiety about, Mathematics

Students rated how well they would perform if asked to solve a number of mathematical tasks (e.g., calculating the petrol consumption rate of a car) using a 4-point scale ranging from 'very confident' to 'not at all confident'. Students were grouped into low, medium, and high 'self-efficacy in mathematics' categories, based on the average degree of confidence they reported in their ability to solve the problems (Table 6.3).

Table 6.3 Mean Combined Mathematics Scores of Students in Ireland by Perceived Self-efficacy in Mathematics

Level	Percent of Students	Mean Mathematics Score
Low	30.4	450.9
Medium	38.9	502.8
High	29.0	559.4
No response	1.6	465.7
All available cases	98.4	503.4

Higher reported self-efficacy in mathematics is associated with higher achievement scores in mathematics. There is a large difference (just over 108 points) between the mean scores of students with high and low self-efficacy in mathematics, in favour of those with high self-efficacy. At international level, a composite measure of self-efficacy was constructed with an OECD mean of zero and a standard deviation of 1. Ireland's mean mathematics self-efficacy score (-0.03) is not significantly different from the OECD country average of zero, and is significantly higher than the mean self-efficacy scores of students in some high-scoring countries in mathematics such as Korea (-0.42) and Japan (-0.53). Male students scored significantly higher on self-efficacy than females in all countries, including Ireland.

Students were asked to rate their anxiety about mathematics achievement by responding to statements such as 'I get very nervous about doing mathematics problems'. As in the case of self-efficacy, students were categorised into low, medium, and high groups based on their aggregate responses across several statements. Students in the low anxiety group obtained the highest mean mathematics score (Table 6.4). The difference between students in the low and medium groups is moderate (34 points), while there is a large difference between the low and high groups (69 points). At the international level, an anxiety about mathematics composite measure, with an OECD mean of zero, and a standard deviation of zero, was constructed. In all countries except Poland and Serbia, male students reported significantly lower levels of anxiety than female students. In Ireland, the difference (-0.27) is about the same as the OECD average difference (-0.25).

Table 6.4 Mean Combined Mathematics Scores of Students in Ireland, by Level of Anxiety about Mathematics

Level	Percent of Students	Mean Mathematics Score
Low	30.7	536.8
Medium	39.8	502.6
High	27.7	468.1
No response	1.8	459.9
All available cases	98.2	503.6

School characteristics and performance

School Size

Irish schools were categorised according to whether they were large (81 or more 15-year olds enrolled), medium (41-80) or small (1-40). Students in large schools significantly outperformed students in medium-sized schools (mean scores are 509.5 and 491.5, respectively). While there was a large difference between the mean scores of students in large and small schools (mean scores = 509.5 and 471.2, respectively), the difference is not statistically significant.¹

School Type/Sector

Schools in Ireland were categorised as being in the secondary, community/comprehensive, or vocational sector (Table 6.5). There was a 40 point difference in mean mathematics achievement favouring students attending secondary schools over students attending vocational schools. Students in secondary schools also significantly outperformed students in community/comprehensive schools by an average of 17 points.

¹ This arises because of the large standard error (uncertainty) associated with the mean score for small schools.

Table 6.5 Mean Combined Mathematics Scores of Students in Ireland, by School Sector

School Type	Percent of Students	Mean Mathematics Score
Secondary	61.0	514.4
Community/Comprehensive	17.3	497.6
Vocational	21.7	474.4
All cases	100.0	502.8

School Socioeconomic Status

In Ireland, schools were categorised according to whether or not they were in the Department of Education and Science Disadvantaged Area Scheme. Students in schools designated as disadvantaged achieved a mean score that was 35 points lower than the mean score of students in non-designated schools (Table 6.6).

Table 6.6 Mean Combined Mathematics Scores of Students in Ireland Attending Designated Disadvantaged and Non-Designated Schools

Disadvantaged Status	Percent of Students	Mean Mathematics Score
Designated	25.4	477.0
Non-designated	74.6	512.3
All cases	100.0	500.3

In each school, the percentage of 15-year old students entitled to the Junior Certificate fee waiver was weighted by the number of students in the school who took the Junior Certificate Examination in 2002 or 2003. Each student was then assigned the value of this variable for his or her school. Students attending schools with high proportions of fee-waiver recipients performed significantly less well on the combined mathematics scale than students attending schools with medium or low proportions of recipients (Table 6.7). The difference in mean achievement between students in high fee-waiver schools (i.e., schools serving mainly low SES students) and students in low fee-waiver schools (serving mainly high SES students) was large (60 points).

Table 6.7 Mean Combined Mathematics Scores of Students in Ireland, by Percentage in School Entitled to a Junior Certificate Examination Fee Waiver

Percent Receiving Fee Waiver	Percent of Students	Mean Mathematics Score
Low	32.8	531.2
Medium	34.2	505.9
High	33.0	471.4
All cases	100.0	502.8

School Disciplinary Climate in Mathematics Classes

Students were asked to rate how often each of five events occurred during mathematics classes, including: 'There is noise and disorder', and 'Students don't listen to what the teacher says'. An overall measure of school disciplinary climate was formed by combining students' responses to such items, and averaging them at the school level. Each student was then assigned the disciplinary climate score corresponding to his/her school. Students in schools with high (positive) disciplinary climate scores significantly outperformed students in schools with medium and low disciplinary climate scores (Table 6.8).

Table 6.8 Mean Combined Mathematics Scores of Students in Ireland, by School-level Disciplinary Climate in Mathematics Classes

Disciplinary Climate	Percent of Students	Mean Mathematics Score
Low	29.2	482.0
Medium	40.4	504.6
High	28.3	525.7
No response	2.1	452.1
All cases	97.9	504.0

It might be noted that disciplinary climate in mathematics classes in Ireland was broadly similar to the OECD average in some respects, and marginally better in others. For example, 32% of students in Ireland reported that there was noise and disorder in mathematics classes, compared to an OECD average of 36%, while 25% said that ‘the teacher had to wait a long time for students to quieten down’, compared to an OECD average of 32%.

Variation between schools in achievement outcomes

In a study such as PISA, where students are clustered in schools, the total variation in achievement can be divided into two components: differences between schools, and differences within schools. PISA did not gather information on particular classes within schools. Therefore, individual differences within schools reflect the variation both between classes and between students. In PISA 2003, 16.7% of the overall variation in achievement in mathematics in Ireland is attributable to differences between schools, compared to an OECD average of 32.7% (Figure 6.2). This suggests that the Irish educational system is relatively uniform at the school level with respect to mathematics achievement. By contrast, the USA has a between-school variance value of 25.8%, while Germany has a value of 52.4%. Large between-school variation is likely to reflect a high degree of selectivity of students into schools. Major sources of variation in the Irish school system are more likely to be found within schools, at the classroom level (e.g., through streaming of students), or among variables corresponding to individual students.

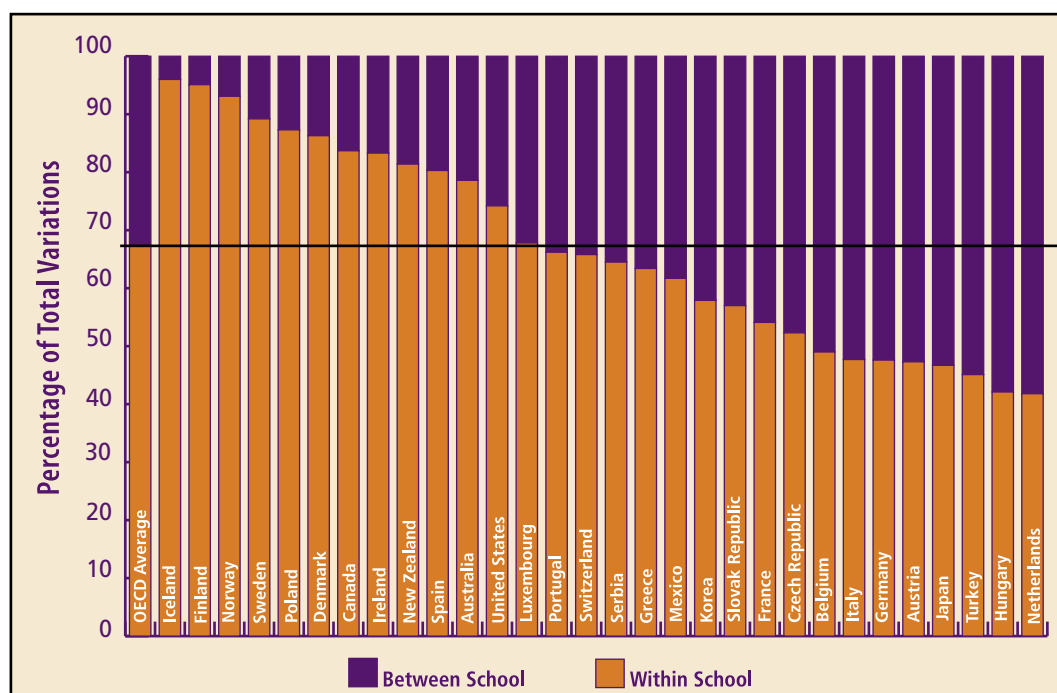
Figure 6.2 Proportions of Between- and Within-School Variation in Mathematics – Ireland and OECD/Partner Countries

Figure 6.2 shows that between-school differences in mathematics achievement in Ireland are fairly modest, at least by OECD standards. Moreover, the impact of school-level SES on school-level achievement in mathematics in Ireland is close to the OECD average (OECD, 2004, Figure 4.13). One can infer from this that, while there are undoubtedly differences in average achievement between schools in Ireland with varying levels of SES intake, those differences are not as pronounced as in several OECD countries, in particular the Czech Republic, Germany and Belgium. Moreover, even high-scoring countries such as Finland show significant associations between school socio-economic status and school-level achievement.

Explaining performance on PISA mathematics

Several of the variables associated with achievement are interrelated. For example, student and school socioeconomic status may be related if lower-SES students attend lower-SES schools. The relationship between mathematics achievement and a range of variables can be examined simultaneously, using a procedure called multi-level modelling. Key variables are selected and then the amount of achievement each variable predicts is estimated, controlling for other variables that are also present. The final model for PISA 2003 combined mathematics reported in Cosgrove et al. (2005) contained two school-level variables (disciplinary climate and socioeconomic status) and eight student-level variables (gender, socioeconomic status, lone-parent status, number of siblings, number of books in the home, home educational resources, frequency of absence from school, and current grade level). The model explained 78.8% of between-school variance in achievement, and 29.6% of within-school variance (i.e., variance at the class and student levels). The model confirmed the effects of school- and student-level socioeconomic status, as well as disciplinary climate, on achievement.

The model included an interaction between number of books in the home and frequency of absence from school. The interaction shows that, in homes where there are up to 25 books, differences in expected scores for students with no absences, and for one, are relatively small, compared with differences for 3 or more absences (Table 6.9). As number of books increases, achievement differences between levels of absence are larger.

Table 6.9 Score Contributions to the Mathematics Achievement of Students in Ireland for Books in the Home by Absence from School

Absence	Book Index (Number of Books in the Home)					
	0 to 10 (1)	11-25 (2)	26-100 (3)	101-200 (4)	201-500 (5)	500+ (6)
None	0.00	26.86	42.57	53.71	62.36	69.42
1-2	3.53	21.64	32.24	39.76	45.59	50.35
3 or more	-16.53	-0.12	9.48	16.29	21.57	25.89

Earlier in this chapter, it was noted that students in Fourth/Transition year did significantly better in mathematics than students in Fifth year. In the context of the model, the difference in predicted scores for students in Fourth and Fifth years (relative to those in Third), was negligible, indicating that other variables in the model (such as socioeconomic status and home educational resources) explained the difference.

Self-efficacy in mathematics and anxiety towards mathematics were not considered for inclusion in the model since their interrelationship with current achievement is such that they may be considered joint outcomes of learning rather than predictors.

Chapter Highlights

- Male students in Ireland achieved significantly higher scores than females on the combined mathematics scale and on all four content scales. The largest difference was observed on the Space & Shape scale. The pattern of gender differences is in line with other OECD countries.
- Students in dual-parent households outperformed students in lone-parent households, while students with no siblings did less well than students with one or two siblings.
- Students with a desk for study at home, a quiet place to study, and books to help with their schoolwork significantly outperformed students who did not have one or more of these resources. Students with more than 100 books at home outperformed students with lower numbers.
- Students who are more confident about their mathematics ability, and less anxious about mathematics, performed better than students who were less confident and more anxious. Male students were more confident and less anxious than females.
- Students with full attendance in the two weeks prior to the PISA assessment significantly outperformed students with one or more absences.
- Students at higher levels of SES (based on their parents' occupations) significantly outperformed students with average and lower levels.
- Students attending schools not designated as disadvantaged, schools with a low level of Junior Certificate fee waivers, secondary schools, and schools with high levels of disciplinary climate in mathematics classes significantly outperformed students in other school categories.
- Between-school variation in achievement in Ireland (17%) was low relative to the OECD country average (33%), indicating a more even spread of achievement in Ireland than in countries with higher levels of selectivity of students into schools.
- A multi-level model of achievement in mathematics confirmed the independent contributions to mathematics achievement of school- and student-level SES, school-level disciplinary climate in mathematics classes, gender, home educational resources, and family structure. The model explained 79% of variation (differences) in achievement between schools, and 30% of the variation within schools.

7 Do teachers in Ireland teach PISA mathematics?

This chapter is based on a questionnaire that was administered to mathematics teachers of students in Ireland who participated in PISA 2003.

Teachers' backgrounds

Two-thirds of the 1273 mathematics teachers in schools participating in PISA 2003 returned completed questionnaires. Of these, 59% were female and almost all were born in Ireland. Teachers reported a mean of 15.9 years' teaching experience in mathematics and 13.6 years teaching in the school they were in at the time of PISA. Just under 90% of teachers were working full-time. Almost all teachers held a bachelor's degree, while 88% had a Higher Diploma in Education. Master's and doctoral degrees were less common with just one in eight holding one or more of these qualifications. The majority of teachers with a bachelor's degree indicated that their degree included a specific mathematics discipline; among those taking a Higher Diploma in Education, almost a third said it included a mathematics education component.

Table 7.1 Percentage of Mathematics Class Time Spent at Various Activities, by Year Level/Programme

Activity	1st, 2nd, 3rd year	5th, 6th year
Administration (e.g., roll call)	4.1	4.0
Reviewing homework	17.7	18.2
Presenting new material	23.8	25.7
Explaining mathematical concepts and procedures (whole class or individuals)	15.0	15.3
Having the students practise routine mathematical operations	15.6	14.7
Having the students solve routine problems	12.6	13.0
Having the students practise transfer of mathematical knowledge to solving problems in real-world situations	4.6	4.1
Dealing with student behaviour	6.0	4.1
Other	0.6	0.9
Total	100	100
Total number of respondents = 663 out of a total of 725 respondents who taught Junior Cycle students at the time of PISA 2003; and 541 to 584 out of a total of 661 respondents who taught Senior Cycle students at the time of PISA 2003. The percentages are based on the numbers of teachers teaching the relevant cycle at the time of PISA 2003 rather than the grand total of 856 teachers.		

Activities during class time

Teachers were asked to indicate the typical percentage of time spent on various activities in Junior and Leaving Certificate mathematics classes (Table 7.1). Homework review took up 18% of class time at both levels. Practising routine mathematical operations and solving routine problems took up just over a quarter of class time, while less than 5% of class time was spent on having students practise transfer of mathematical knowledge to solving problems in real-world situations (a key element of PISA mathematics).

Teachers' views of mathematics as a subject

Table 7.2 shows the percentages of teachers expressing agreement/disagreement with seven statements about the nature of mathematics/mathematics education. There was a high rate of agreement that some students have a natural talent for mathematics, while others do not. Furthermore, most teachers agreed or strongly agreed that more than one representation should be used in teaching a mathematics topic. Given the aims and focus of PISA mathematics, it is noteworthy that 6 in 10 teachers felt that an understanding of how mathematics is used in the real world is needed to be good at mathematics at school.

Table 7.2 Cross-Classified Percentages of Teachers' Agreement/Disagreement with Seven Statements about the Nature of Mathematics as a Subject

	Strongly Agree	Agree	Disagree	Strongly Disagree	Total
Mathematics is primarily an abstract subject	3.1	33.2	51.6	12.1	100
Some students have a natural talent for mathematics and others do not	27.9	64.5	7.3	0.3	100
If students are having difficulty, an effective approach is to give them more practice by themselves during the class	9.2	55.5	30.8	4.5	100
More than one representation (picture, concrete material, symbol set, etc.) should be used in teaching a mathematics topic	34.4	60.0	5.4	0.2	100
To be good at mathematics at school, it is important to understand how mathematics is used in the real world	13.4	46.4	36.8	3.4	100
Mathematics is a difficult subject for most students	3.0	34.3	58.6	4.1	100
A good understanding of mathematics is important for other subjects	7.3	58.4	32.1	2.2	100
Total number of respondents = 823 to 850 (varies by item).					

Homework and assessment

Almost all teachers reported assigning homework in most or all lessons to Junior Cycle students at Higher and Ordinary level while 4 out of 5 teachers reported giving homework to Foundation level students with the same frequency. Further, approximately three-quarters of teachers at all levels reported giving their students a quiz or test (other than in-house or mock examinations) at least once a month.

Almost all teachers agreed or strongly agreed that homework is an effective way for students to consolidate class work, and that it helps to monitor students' progress (Table 7.3). However, just over half disagreed or strongly disagreed that they often assigned homework which required application of concepts in novel contexts (a key element of PISA mathematics). Almost two-thirds of teachers also disagreed that it was important to assign project work in mathematics to students

Table 7.3 Teachers' Agreement/Disagreement with Six Statements About Homework

	Strongly Agree	Agree	Disagree	Strongly Disagree	Total
Homework is an effective way for students to consolidate what has been covered in class	72.2	27.1	0.6	0.1	100
I often assign homework that requires students to apply knowledge of concepts in novel contexts	5.6	40.8	50.8	2.8	100
Regular homework assignments help to monitor students' progress	42.7	53.8	3.0	0.5	100
Homework is a good way of identifying students' weaknesses	32.3	55.3	11.3	1.1	100
The main purpose of homework is to prepare students for the State Examinations	6.6	33.9	53.2	6.3	100
It is important to assign project work in maths to students	4.7	32.2	57.7	5.4	100
Total number of respondents = 821 to 852 (varies by item).					

Instructional content and emphasis at Junior Cycle

Teachers were asked to show the degree of emphasis they placed on various aspects of the Junior Certificate mathematics syllabus and of the PISA 2003 mathematics framework on a 4-point scale, with higher values representing a higher level of emphasis ('a lot'=4; 'some'=3; 'a little'=2; 'none'=1). The outcomes are given in Tables 7.4 to 7.6.

Emphasis Placed on Aspects of the Junior Certificate Syllabus

Teachers reported placing most emphasis at Higher level on developing the application of mathematical knowledge, and at Ordinary and Foundation levels on teaching recall of basic facts. In working with students at all syllabus levels, developing an awareness of the history of mathematics and its role in culture and society and developing an appreciation of the history of mathematics received the least emphasis.

Table 7.4 Mean Levels of Emphasis Given by Teachers to Eight Objectives Relating to Junior Certificate Mathematics, by Syllabus Level

Objective (Junior Cert. Syllabus)	Higher	Ordinary	Foundation
Teaching students to recall basic facts	3.5	3.5	3.4
Teaching instrumental understanding	3.2	3.4	3.3
Developing relational understanding	3.5	3.3	3.0
Developing application of mathematical knowledge	3.7	3.2	2.8
Developing skills of analysis	3.2	2.6	2.3
Developing creativity and communication skills in mathematical thinking	2.7	2.4	2.1
Developing an appreciation of mathematics	2.7	2.6	2.5
Developing an awareness of the history of mathematics and its role in culture and society	1.9	1.9	1.8
Note. Ratings based on a 4-point scale with higher values representing a higher emphasis ("a lot"=4, "some"=3, "a little"=2, "none"=1).			

With regard to preparation for the Junior Certificate Examination, highest emphasis at all three syllabus levels was given to attempting sample questions both in class and at home (Table 7.5). High emphasis was also placed on familiarising students with timing and format.

Table 7.5 Mean Levels of Emphasis Given to Four Aspects of Preparation for the Junior Certificate Mathematics Examination, by Syllabus Level

Aspect of Preparation	Higher	Ordinary	Foundation
	Mean	Mean	Mean
Attempting questions from sample examination papers in class	3.7	3.8	3.9
Assigning questions from sample examination papers for homework	3.7	3.7	3.8
Familiarising students with the format and timing of the examination	3.4	3.5	3.7
Advising students on appropriate choice of questions in the examination	2.7	2.9	3.2
Note. Ratings based on a 4-point scale with higher values representing a higher emphasis ("a lot"=4, "some"=3, "a little"=2, "none"=1).			

Emphasis Placed on Aspects of PISA Mathematics

Table 7.6 shows the degree of emphasis placed on aspects of the four PISA mathematics content scales by teachers of Junior Cycle students. On Space & Shape, the skill of recognising shapes and patterns received the highest emphasis at all three syllabus levels. Other aspects, such as representing 3-D objects in two dimensions, and navigating through space, received comparatively little attention. On Change & Relationships, the skills of mathematical modelling of functions and translating one representation into another received more emphasis at Higher and Ordinary levels than representing change/relationships in different formats. All aspects of Change & Relationships received less emphasis at Foundation than at Higher and Ordinary levels.

All aspects of Quantity, with the exception of representing numbers in various ways, have mean emphasis ratings between 'a lot' and 'some' at all syllabus levels. In the Uncertainty

subdomain, data analysis and data display were fairly strongly emphasised at Higher and Ordinary levels. Other aspects, such as understanding the concepts of variability and uncertainty, understanding simple random sampling, and applying probability and inference, were not strongly emphasised at any level, reflecting the fact that probability is not on the Junior Certificate syllabus.

Table 7.6 Mean Levels of Emphasis Given by Teachers to Aspects of Four PISA Mathematics Content Areas, by Junior Certificate Syllabus Level

Aspect of Content Area	Higher	Ordinary	Foundation
	Mean	Mean	Mean
Space & Shape			
Recognising shapes and patterns	3.11	3.21	3.21
Representing three-dimensional objects in two dimensions	2.52	2.51	2.26
Navigating through space	1.92	2.10	1.89
Navigating through constructions or shapes	1.87	1.78	1.53
Change & Relationships			
Recognising types of change/relationship	2.77	2.38	1.98
Understanding types of change/relationship	2.68	2.25	1.92
Mathematical modelling of functions	3.51	3.26	2.70
Representing change/relationship in different formats	3.19	2.88	2.56
Translating one representation of change/relationship to another	3.46	3.42	3.06
Quantity			
Developing number sense	3.38	3.53	3.59
Demonstrating an understanding of magnitude	3.13	3.23	3.28
Demonstrating an understanding of the meaning of mathematical operations	3.46	3.54	3.36
Developing efficient computational skills	3.39	3.43	3.37
Developing mental arithmetic and estimation skills	3.24	3.19	3.05
Representing numbers in various ways	2.92	2.90	2.74
Uncertainty			
Understanding the concepts of variability and uncertainty	2.50	2.35	2.05
Data analysis	3.32	3.21	2.97
Data display	3.17	3.28	2.94
Understanding the concept of simple random sample	2.27	2.12	1.87
Understanding the concepts of probability and inference	1.85	1.69	1.41
Applying the concepts of probability and inference	1.72	1.63	1.41
Note. Ratings based on a 4-point scale with higher values representing a higher emphasis ("a lot"=4, "some"=3, "a little"=2, "none"=1).			

These outcomes show that coverage of aspects of Space & Shape, on which students in Ireland did poorly in PISA 2003, is limited. They also indicate that several important aspects of Quantity are emphasised at least to some extent. It may be that, despite this coverage, students' unfamiliarity with the contexts in which Quantity items were presented in PISA, and the level of relational understanding required by many such items, meant that students in Ireland only performed at an average level in this content area. Although some aspects of Change & Relationships received little emphasis, others such as mathematical modelling of functions received a lot. This, in turn, may have contributed to the above-average performance of students in Ireland on this domain. The strong performance of students in Ireland on Uncertainty can be explained in part by the relatively strong emphasis placed by teachers on data analysis and data display. It is clear that the curriculum in Ireland (and perhaps in other countries also) does not place much emphasis on the probability aspects of uncertainty at this level.

Chapter Highlights

- Responding teachers had, on average, 16 years teaching experience in mathematics.
- At both Junior and Leaving Certificate levels, 19% of class time was spent on reviewing homework, and 28% on students practising routine mathematical operations and solving routine problems. Just 4% of time was spent on transferring mathematical knowledge to solving problems in real-world situations.
- Almost all teachers agreed that more than one representation should be used in teaching a mathematics topic.
- Forty percent of teachers disagreed with the view that, to be good at mathematics in school, it is important to understand how mathematics is used in the real world.
- Teachers confirmed that they did not teach key aspects of PISA Space & Shape, including representing 3-D objects in two dimensions, and 'navigating through space'.

8 What can we learn from PISA mathematics?

This chapter considers the outcomes of the PISA 2003 mathematics study from three perspectives: (i) the performance of students in Ireland in an international context; (ii) links between PISA mathematics and Junior Certificate mathematics; and (iii) implications of PISA 2003 for teaching and learning mathematics.

Performance of students in Ireland

Overall Performance

The overall performance of students in Ireland on PISA 2003 mathematics was close to the OECD country average. Students in Ireland ranked 17th of 29 OECD countries, and 20th of 40 participating countries. This performance may be contrasted with Irish students' performance in PISA 2003 reading literacy and science, for which mean scores were above the corresponding OECD country averages. Several factors may explain the performance of students in Ireland. These include the relative unfamiliarity of students with some of the contexts and processes (competencies) underlying PISA mathematics (see next section), and the motivation of students in a low-stakes assessment such as PISA (though there is no evidence that this was a problem for reading literacy or science). It might also be noted that there are substantial aspects of the Junior Certificate syllabus (including Algebra and Geometry) that are assessed only to a small extent or are not assessed at all in PISA mathematics. This implies that students in Ireland did not have an opportunity to display their full range of knowledge in these aspects of mathematics.

Performance on the Four Subscales

The performance of students in Ireland varied across the PISA mathematics content areas. Mean scores were above the OECD average in two areas (Change & Relationships and Uncertainty), not significantly different on one (Quantity), and significantly below it on one (Space & Shape). The below-average performance in Space & Shape may be explained in terms of differences between PISA Space & Shape, which tends to focus on patterning and recognition of shapes in different representations and dimensions, and Euclidean geometry, as represented in the Junior Certificate syllabus. Although teachers in Ireland confirmed that they emphasised Quantity in their teaching, students may have struggled on some of the Quantity items because of the contexts in which they were embedded, or because they were not used to applying the higher-level competencies assessed in such items. Indeed, the finding by Close and Oldham (2005) that the vast majority of items on the 2003 Junior Certificate mathematics examination at all three syllabus levels fell into the lowest PISA competency cluster (i.e., Reproduction), suggests that students in Ireland may have had limited experience with the higher-order mathematical processes required by PISA (i.e., Connections, Reflection). The above average performance of students in Ireland on Change & Relationships may reflect the breadth of the Junior Certificate curriculum in Ireland, as items in this subdomain were distributed over several Junior Certificate content areas, including Algebra, Statistics, Functions and Graphs, and Applied Arithmetic & Measure. The emphasis placed by teachers on mathematical modelling and functions at Junior Certificate level may also have contributed. Finally, while Ireland's strong performance on Uncertainty can be explained in part by its overlap with Statistics in the Junior Certificate syllabus, and by the relatively strong emphasis that teachers place on this aspect of the syllabus, it is unclear why students did well on items dealing with probability and inference, given the absence

of these topics from the Junior Certificate syllabus. Perhaps it can be attributed to informal knowledge acquired outside school, though Ireland's performance may have been influenced by students in the Fourth and Fifth years, who may have studied aspects of probability and inference. The relatively strong performance of students in Ireland on Uncertainty may also be explained, at least in part, by the performance of students in other countries.

Differences Between Low and High Achievers

Lower-achieving students in Ireland (those scoring at the 10th percentile) obtained a score that was 34 points higher than the corresponding OECD country average. On the other hand, higher-achieving students (those scoring at the 90th percentile) achieved a score that was 14 points lower than the OECD average score at that benchmark. Hence, while low-achieving students in Ireland did reasonably well, higher achievers underperformed relative to students elsewhere. It is unclear if the performance of high-achieving students in Ireland is attributable to lack of opportunity to engage in the higher-level tasks embedded in PISA, as they engage with mathematics at school and in exam contexts, or if other factors, such as the low-stakes nature of the PISA assessment, may have been implicated.

The fact that 17% of students in Ireland achieved at or below Level 1 (compared to an OECD average of 21%), while positive in some respects, can also be interpreted in the context of the OECD view that students scoring below Level 2 are unlikely to have the knowledge and skills in mathematics that are needed for further study and for future life needs. Following this logic, it can be concluded that 1 in 6 students in Ireland is poorly prepared for their future mathematics needs as students and citizens.

Relationships among school and student characteristics and achievement

Gender

Male students in Ireland outperformed females on the combined mathematics scale. The difference was also found in the multi-level model of achievement in mathematics, when other variables that might explain gender differences were controlled for. The overall difference is remarkable in light of the stronger performance of female students in mathematics in the Junior Certificate examination in recent years. The reasons for this may relate to the different functions of the two assessments, the use in PISA of a sizeable number of multiple-choice items (on which males tend to do better), and, perhaps, a greater propensity among male students to take risks when attempting the PISA items. The strong performance of male students relative to female students in Ireland on Space & Shape items is noteworthy.

Home Educational Resources

The finding that the number of books in a student's home is a predictor of performance, even when the effects of other variables such as socioeconomic status are held constant, is interesting. It may be that students who live in homes with large numbers of books experience different levels of support, and different expectations in relation to doing well in school, than students in homes that lack books.

Disciplinary Climate in Mathematics Classes

It is noteworthy, in the context of the multi-level model of mathematics achievement, that the average level of disciplinary climate in mathematics classes in a school is significantly associated with students' achievement in mathematics. While, on the surface, variables such as the noise level in mathematics classes, the attentiveness of fellow students, and focus during lessons are important, it may also be the case that disciplinary climate represents an ethos towards doing well in mathematics that cannot be explained solely in terms of school- or student-level socioeconomic status.

Socioeconomic Status

Students attending schools designated as disadvantaged and schools with large numbers of students in receipt of a fee waiver for the Junior Certificate examination performed significantly less well on the PISA mathematics assessment than students attending other schools. Moreover, the multilevel model of achievement in mathematics indicated that both school- and student-level socioeconomic status contribute to achievement in mathematics, even after controlling for other related variables (e.g., home educational resources, absence from school, number of books in the home). This implies that, on average, low-SES students are more at risk of low achievement, particularly when they attend schools in which large numbers of students are also socioeconomically disadvantaged. While it is acknowledged that the impact of socioeconomic status on achievement at both school and individual levels in Ireland is close to the OECD average impact, it is nevertheless important to promote higher levels of achievement among socioeconomically disadvantaged students, not least because of the postulated importance of mathematics to later education and successful functioning in society.

Variation Between Schools in Achievement Outcomes

In Ireland, just 17% of the variation in mathematics achievement was attributable to differences between schools. This compared favourably with the OECD average of 33%, and can be interpreted as indicating that schools in Ireland are more equitable in terms of mathematics performance than schools in most OECD countries. This may arise because a common mathematics curriculum is taught in almost all schools. One can only assume that differences between school types (for example, between secondary and vocational schools) would be even greater if more differentiated curricula were implemented.

PISA and the Junior Certificate syllabus and examination

The analyses in this guide show clear differences between PISA mathematics and the Junior Certificate mathematics syllabus and examination. For example, it was expected that, since students are not used to solving problems embedded in real-world contexts, they would be unfamiliar with many of the contexts in which PISA items are presented. Furthermore, since, unlike PISA, most items on the Junior Certificate mathematics examination tend to call on lower-level competencies, it was thought that students would be at a disadvantage in responding to items requiring the application of higher-level competencies, such as those assessed by the Connections and Reflection items.

The issue of whether to move toward a PISA-style mathematic curriculum at post-primary level is one that is currently being considered by the NCCA. Conway and Sloane (2005) point out that such a decision is not trivial, and note the absence of formal geometry (as defined in the Junior Certificate syllabus) and formal trigonometry from PISA. They also note, however, that an increased emphasis on problem-solving, and on presenting problems in real-world contexts would be consistent with the aims of the current primary school mathematics curriculum, as well as current constructivist interpretations of knowledge building. Ultimately, it may be a case of adjusting the current Junior Certificate syllabus and examinations to address some of the apparent shortcomings identified by PISA and other studies, while at the same time retaining the most important content.

PISA and the teaching of mathematics

Lyons et al.'s (2003) observational study suggests that, up until recently, at least in some schools, much of the teaching and learning in Junior Certificate mathematics classes was didactic with relatively little emphasis on the explanation of concepts, and few opportunities for teachers to engage in problem-solving. PISA provides a different framework for teaching and learning mathematics that is worth examining more closely.

While the lesson ideas in *Junior Certificate Mathematics: Guidelines for Teachers* (DES/NCCA, 2002) include several useful activities for engaging students in real-world problems, and promoting aspects of relational thinking, teachers may also want to consider ways in which specific aspects of PISA could be applied in mathematics classes. The text box provides some suggestions for accomplishing this. Teachers will note that some of the suggestions, including those relating to the use of vocabulary and language in mathematics, and extraction of mathematical information for real-world problems, may also be found in the Chief Examiners' reports on Junior Certificate Mathematics examinations in 2003 (SEC, 2003).

Suggestions for Applying the PISA Approach to Teaching and Learning Mathematics

- Emphasise a more interactive approach to teaching mathematics, in which students are engaged in discussing problems, both before they are solved, and afterwards. Discussion should focus on identifying the mathematics needed to solve a problem, and on communicating students' reasoning after it has been solved.
- Emphasise the full range of cognitive competencies (processes) during teaching. The over-emphasis on reproduction in classrooms and in examinations means that many students may not get an opportunity to apply higher-level competencies such as Connecting and Reflecting. It is likely that the application of these competencies by students at all levels of ability will result in greater conceptual understanding and more independence in solving problems.
- Implement a better balance of context-free questions and questions that are embedded in real-world contexts. Many of the questions in current textbooks and examination papers are context-free. While such items play an important role in developing basic mathematics skills, it is also important to provide students with opportunities to engage with real-world problems. Such engagement serves to make mathematics more relevant for them, and provides them with opportunities for developing a broader range of mathematical competencies.
- Emphasise more use of language in mathematics classes. A potential drawback of the PISA approach is the need for students to call on language skills (including reading and writing) as they engage with mathematics problems. Teachers can support these processes by engaging students more often in discussions about how to solve problems, and how the solutions of problems can be applied in real-world contexts.
- Help students to develop mathematical knowledge in the context of solving problems. This can be achieved in part by providing students with real-world mathematics problems and by discussing with them the mathematics involved and the ways in which this mathematics can be applied to other problems.
- Provide higher-achieving students with more challenges in mathematics. PISA 2003 suggests that higher-achieving students in Ireland could be challenged to a greater extent. Notwithstanding the requirement to prepare such students for the Junior Certificate Mathematics examination, it would be advantageous to challenge them to solve more complex PISA-style mathematics items which would require them to extract mathematical information from real-world problems.
- Transition year may provide an opportunity to engage students at all levels of ability in solving the types of real-world mathematics problems found in PISA.

References

- Close, S. (2006). The junior-cycle curriculum and the PISA mathematics framework. *Irish Journal of Education*, 37, 53-78.
- Close, S., & Oldham, E. (2005). Junior cycle mathematics and the PISA mathematics framework. In S. Close, T. Dooley & D. Corcoran (Eds.), *Proceedings of the First National Mathematics in Education Conference*, Dublin. Accessed online at: http://www.spd.dcu.ie/main/academic/education/documents/Proceedings_000.pdf
- Close, S., Oldham, E., Hackett, D., Dooley, T., Sheil, G., & O'Leary, M. (2003). *A study of the effects of calculator use in schools and in the certificate examinations. Summary report for phase 1*. Dublin: St. Patrick's College, Drumcondra.
- Conway, P.F. & Sloane, F. (2005). *International trends in post-primary mathematics education: Perspectives on learning, teaching and assessment*. Dublin: National Council for Curriculum and Assessment.
- Cosgrove, J., Shiel, G., Sofroniou, N., Zastrutzki, S., & Shortt, F. (2005) *Education for life: The achievements of 15-year-olds in Ireland in the second cycle of PISA*. Dublin: Educational Research Centre.
- DES/NCCA. (Department of Education and Science/National Council for Curriculum and Assessment). (2000). *Junior Certificate mathematics syllabus (Higher, Ordinary and Foundation level)*. Dublin: Stationery Office.
- DES/NCCA. (2002). *Mathematics Junior Certificate. Guidelines for teachers*. Dublin: Stationery Office.
- Freudenthal, H. (1973). *Mathematics as an educational task*. Dordrecht: Kluwer Academic.
- Freudenthal, H. (1981). Major problems in mathematics education. *Educational Studies in Mathematics*, 12, 133-150.
- Lyons, M., Lynch, K., Close, S., Sheerin, E., & Boland, P. (2003). *Inside classrooms: The teaching and learning of mathematics in social context*. Dublin: Institute of Public Administration.
- NCCA (National Council for Curriculum and Assessment). (2006). *Review of mathematics in post-primary education: Report on the consultation*. Retrieved January 2007 from: http://www.ncca.ie/uploadedfiles/mathsreview/Maths_Consult_Report.pdf
- OECD (Organisation for Economic Co-operation and Development). (2003). *The PISA 2003 assessment framework: Mathematics, reading, science and problem solving knowledge and skills*. Paris: Author.
- OECD. (2004). *Learning for tomorrow's world: First results from PISA 2003*. Paris: Author.
- Oldham, E. (2006). The PISA mathematics results in context. *Irish Journal of Education*, 37, 27-52.
- SEC (State Examinations Commission). (2003). *Junior Certificate Examination 2003: Mathematics: Chief Examiners' reports*. Athlone: Author. Retrieved February 2007 from: http://www.examinations.ie/archive/examiners_reports/JCMaths03.pdf

Glossary of Terms

The PISA survey uses specific terms to describe various aspects of assessment. Further, some technical and statistical terms are used in this guide. These are explained in a little more detail here.

Correlation. References are made to the correlation between achievement on PISA mathematics and performance on the Junior Certificate mathematics examination. The correlation is a measure of linear association and should not be interpreted to mean that one variable is the cause of another. Rather, it suggests that they are associated, possibly by connection with other variables. Values of correlations can range from -1 to +1. When a correlation is negative, the increase in one variable is associated with a decrease in the other variable; when it is positive, an increase in one variable is associated with an increase in the other. A value of 0 indicates no association between two variables.

Item Scale Score. PISA is scaled using Item Response Theory. This enables the placement of items and students on the same underlying scale. On the PISA combined mathematics scale, an item scale score of 450 to 550 indicates that the item has average difficulty across OECD countries. An item score that is less than 450 indicates that the item is relatively easy, while an item score that is greater than 550 indicates that the item is relatively difficult.

Major Domain, Minor Domain. In PISA, the areas of assessment are referred to as domains. In PISA 2003, the main focus was on mathematics, and it is referred to as the major domain. Reading and scientific literacy, as well as cross-curricular problem solving, received less emphasis and are referred to as minor domains. Just over half of participating students attempted items from each of these minor domains, and there were fewer items compared to mathematics.

Percentile. A percentile rank is the percentage of scores in a distribution that are at or below a given score. For example, if a student in Ireland achieved a score of 641 on PISA mathematics, his/her score would be at the 90th percentile, indicating that he/she did as well as, or better than, 90% of 15-year olds in Ireland on the test.

Proficiency Level. Performance on the PISA 2003 combined mathematics scale and four mathematics content area scales can be interpreted with reference to proficiency levels. On each of these scales, Level 1 extends from 359 to 420 points; Level 2 from 421–482; Level 3 from 483 to 544; Level 4 from 545 to 606; Level 5 from 607 to 668; and Level 6 from 669 upwards. An additional level, called 'Below Level 1, covers scores that are less than 359. All students within a level are expected to get half of the items at that level correct (and fewer than one-half of item at higher levels correct). A student scoring at the bottom of a proficiency level has a .62 probability of answering the easiest items at that level correctly, and a .42 probability of answering the most difficult items correctly. A student scoring at the top of a level has a .62 probability of getting the most difficult items right, and a .78 probability of getting the easiest items right. Students below Level 1 are expected to respond correctly to fewer than 50% of Level 1 items. Since PISA is scaled using Item Response Theory methodology, item scores are on the same scale as student scores. Hence, item scores can also be interpreted in terms of proficiency levels. An item with a scale score of 400 is at Level 1 (indicating that it is relatively easy); an item with a scale score of 500 is at Level 3 (indicating average difficulty); and an item with a scale score of 650 is at Level 5 (indicating high difficulty).

Real-World Mathematics Knowledge. Reference to real-world mathematics knowledge or mathematics literacy reflects the philosophy underlying PISA. Since the focus of PISA is the assessment of outcomes for students who are near the end of compulsory schooling, it is of

interest to find out how well these young adults are equipped for participation in work and wider society as well as future education. Hence, the PISA assessment is not linked directly to school curricula, but reflects the views of educators in participating countries on what young adults need to know to participate in society.

Rotated Booklet Design. PISA used a rotated booklet design. This means that each participating student was given one of 13 possible test booklets at random. Each booklet contained four half-hour blocks of about 15 items (questions, tasks). All booklets contained some mathematics blocks, while 7 of the 13 booklets contained reading blocks and the same number contained science and problem-solving blocks. By linking items that are common across booklets, an equivalent achievement score for mathematics is assigned to each student regardless of the particular booklet attempted. A rotated design is used to obtain broad coverage of the assessment domains (it would not be reasonable to give every student the total number of PISA assessment items).

Standard Deviation. The standard deviation (sd) associated with a score in PISA is an indication of the spread of scores obtained by students in a region, country, or subgroup. It provides a useful way of interpreting the difference in mean scores between groups, since it corresponds to percentages of a normally distributed population. For example, 68% of students in the population have an achievement score that is within one standard deviation of the mean (± 1 sd), and 95% of the population has an achievement score that is within two standard deviations of the mean (± 2 sd). Across the OECD as a whole, 68% of pupils have an achievement score in mathematics between 400 and 600 and 95% of pupils have an achievement score between 300 and 700. In the case of Ireland, which has a mean score of 502.8 and a standard deviation of 85.3, 68% of students' scores fall within the interval 417.5 to 588.1, and 95% score between 332.2 and 673.4. Where international comparisons are made, the OECD value for the standard deviation (100) is used; where comparisons are made between groups within Ireland, the Irish value (85.3) is used.

Statistical Significance. The achievement scores of students are not error-free. They include error due to sampling and measurement procedures. Therefore, statistical tests of association (correlation) and tests for differences between mean scores of groups incorporate this degree of uncertainty due to error. Throughout this guide, correlations and differences between group means are statistically significant when there is a 19 in 20 chance that a difference between groups remains, even after allowing for error, unless otherwise stated. In this guide, we refer to the outcomes of these statistical tests as either 'significant' or 'not significant'.

Variable. A variable is a quantity or attribute that may assume one of a range of values. Outcome variables, in this case student achievements, are related to a number of background or explanatory variables, i.e., characteristics of students, their home and school backgrounds, to highlight differences between subgroups of students based on different quantities of the explanatory variables. The interpretation of these differences helps to identify areas of inequity, and strengths and weaknesses of the education system, and thus to inform policy and pedagogical practice. Variables generally fall into one of two groups. Continuous variables are measured on a scale with a wide range of values. For example, SES was measured on a continuous scale ranging from 16-90. Continuous variables are sometimes constructed by combining responses to several related agree-disagree statements, forming a combined or composite variable. Categorical variables involve classification into discrete values or categories. For example, secondary, vocational, and community/comprehensive schools are categories of the variable school type/sector. Categorical variables can be ordered (such as high, medium, and low SES) or unordered (such as school type).



ISBN: 0-7557-7586-4